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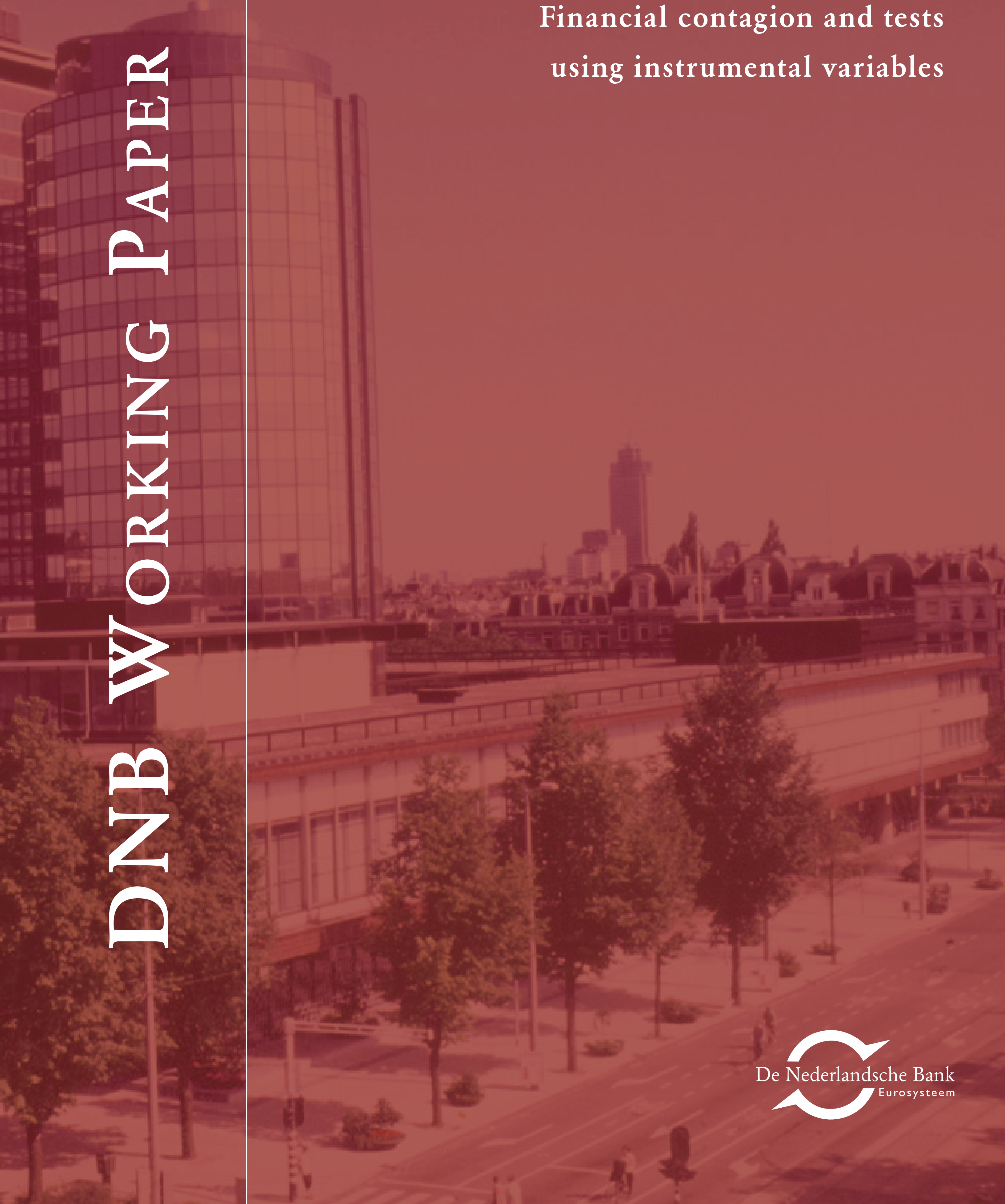

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* Views expressed are those of the author and do not necessarily reflect official positions of De Nederlandsche Bank.

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Financial contagion and tests using instrumental variables

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Abstract This paper considers empirical tests for the contagion of financial crises that address the endogeneity of contagion by using instrumental variable estimation techniques. Two complications in the application to contagion are that the regression model is potentially incoherent and that it contains a parameter that is not identified under the null of no contagion. Monte Carlo experiments suggest that their influence is small in practice with the notable exception of similar tests, where both size and power are affected. An application to stock market data for the UK, USA, and Japan shows that ignoring the endogeneity of contagion leads to highly significant contagion coefficients. However, tests for contagion that take the endogeneity into account result in mixed evidence for financial contagion.

JEL classification C12, G10, G15
Keywords Financial crises, contagion, non-linear simultaneous equation models

1 Introduction

The possibility of the contagion of financial crises has received considerable attention in the recent empirical literature in international finance. A reason is that financial contagion has important implications for policy interventions in financial markets and portfolio diversification of investors. Both crucially depend on the nature of the transmission of shocks between financial markets. While financial markets are interdependent at all times, contagion implies that the correlation between markets increases at times of financial crises. Changes in the transmission of shocks during crises imply that a policy response to a crisis cannot rely on the lessons learned during non-crises times. Furthermore, portfolios constructed based on correla-

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tions during non-crises times would potentially exhibit different properties in crises times, which could lead to considerably larger losses than expected.

A canonical model of contagion has been developed by Pesaran and Pick (2007). On the basis of this model Pesaran and Pick analyse the extant literature on contagion and conclude that panel data analyses, such as those by Eichengreen, Rose and Wyplosz (1996), Kruger, Osakwe and Page (1998), or Kumar, Moorthy and Perraudin (2002), need to take the endogeneity of contagion into account, and that failure to do so may lead to spurious findings of contagion.[1] The endogeneity of contagion requires the use of instrumental variables techniques, and Pesaran and Pick give an example which shows that the correction for endogeneity using two stage least square estimation reduces the evidence for financial contagion.

This paper continues the development of tests of financial contagion that use instrumental variables. The finite sample properties of instrumental variable methods when applied to financial contagion are investigated, and a wide range of instrumental variable methods are considered, including recently developed similar tests. In addition, two features of the canonical model of contagion are addressed that require attention. The first is the incoherence of the canonical model, that is, for certain values of the exogenous variables and the error term the right hand side variables do not imply a unique left hand side variable. While it is not necessary to resolve the incoherence for the identification of the contagion parameter, the efficiency of the estimation are unknown. Monte Carlo experiments investigate this and the results suggest that the effect is small.

The second feature is that the canonical model of contagion contains a parameter that is not identified under the null of no contagion. This is a well known problem in other models such as the threshold auto-regressive model, for which simulation methods have been developed to approximate the distribution of the parameters for inference. Here, the parameter in question is embedded in the endogenous variable, and the simulation methods are therefore invalid. In this paper I use Monte Carlo experiments to assess the impact of the unidentified parameter on inference about contagion.

The instrumental variable techniques are then applied to stock market indices from the UK, the USA, and Japan in order to test whether contagion between these stock markets is present. However, I find that in practice the limited predictability of stock market returns also limits the inference on contagion.

In the next section I discuss the canonical model of contagion and its econometric implications. Section 3 discusses instrumental variables meth-

[1] Another strand of the emprical literature on financial contagion assesses the difference in correlation between markets at tranquil and at crises times. Examples include Forbes and Rigobon (2002), Bae, Karolyi and Stulz (2003), and Corsetti, Pericoli and Sbracia (2005). However, Pesaran and Pick (2007) point out that this approach suffers from a sample selection problem and, therefore, I do not pursue this approach further.

ods and their application to the canonical model. Their finite sample properties are assessed in Section 4. Section 5 applies the methods to the stock market data for the UK, the USA, and Japan. Finally, Section 6 concludes.

2 A canonical model of contagion

Consider the following model of financial contagion introduced by Pesaran and Pick (2007)

$$y_{it} = \boldsymbol{\alpha}_i' \mathbf{x}_{it} + \beta_i \mathcal{C}_{it} + u_{it}, \quad i = 1, 2, \ldots, N, \; t = 1, 2, \ldots, T \tag{1}$$

where i indicates cross-section units such as countries, markets, or assets, t indicates time, y_{it} is a performance indicator, for example a stock market index or an exchange rate market pressure index, $\mathbf{x}_{it}$ is a $(K \times 1)$ vector of predetermined variables, $\boldsymbol{\alpha}_i$ is a $(K \times 1)$ vector of parameters, β_i is a scalar parameter. Define the $(K + 1 \times 1)$ vector $\boldsymbol{\theta}_i = (\boldsymbol{\alpha}_i, \beta_i)$. For simplicity of exposition and without loss of generality I have dropped the common regressor set $\mathbf{z}_t$ included by Pesaran and Pick (2007).

The performance indicator, y_{it}, is assumed to be in a crisis period if it exceeds a given threshold value. A crisis is therefore defined to occur if

$$\mathrm{I}(y_{it} - c_i) = 1,$$

where $\mathrm{I}(A)$ is an indicator function that takes the value of unity if $A > 0$ and zero otherwise, c_i is a crisis threshold. The contagion index, $\mathcal{C}_{it}$, is defined as

$$\mathcal{C}_{it} = \mathrm{I}\left(\sum_{j=1, j \neq i}^{N} \mathrm{I}(y_{jt} - c_j) \right). \tag{2}$$

An alternative definition of contagion would be a weighted average of individual crises indicators. However, as formulation (2) is the definition of contagion commonly found in the literature I will restrict attention to this formulation. The general results carry over to alternative definitions discussed in Pesaran and Pick (2007).

The error term, u_{it}, is assumed to be potentially contemporaneously correlated across i, $\mathbf{u}_t \sim \mathrm{N}(\mathbf{0}, \boldsymbol{\Sigma})$, where $\boldsymbol{\Sigma}$ is a $(N \times N)$ symmetric, positive definite matrix. An example is a common factor structure of the form

$$u_{it} = \gamma_i f_t + \varepsilon_{it}, \tag{3}$$

where γ_i is a scalar parameter, $f_t \sim \mathrm{iid}\,\mathrm{N}(0, 1)$, is the common unobserved factor, and $\varepsilon_{it} \sim \mathrm{iid}\,\mathrm{N}\big(0, \sigma_\varepsilon^2\big)$.

It is apparent that the system of equations in (1) represents a non-linear simultaneous equation model, which is linear in the parameter of interest,

β_i, but non-linear in the endogenous variables. Pesaran and Pick (2007) derive the reduced form for $N = 2$, which is

$$\begin{aligned}
Y_{it} \;=\; & (1+W_{it})\,\mathrm{I}(W_{jt}) && \text{(Region A)} \\
& +(1+W_{it})\,\mathrm{I}(-W_{jt})\ \mathrm{I}(1+W_{jt})\ \mathrm{I}(W_{it}) && \text{(Region B)} \\
& +W_{it}\,\mathrm{I}(-1-W_{jt}) && \text{(Region C)} \\
& +W_{it}\,\mathrm{I}(-W_{jt})\ \mathrm{I}(1+W_{jt})\ \mathrm{I}(-1-W_{it}) && \text{(Region D)} \\
& +Y^{*}_{it}(d_t)\,\mathrm{I}(-W_{it})\ \mathrm{I}(1+W_{jt}) && \text{(Region E)} \\
& \times\,\mathrm{I}(-W_{it})\ \mathrm{I}(1+W_{it})
\end{aligned} \tag{4}$$

where $Y_{it} = \frac{y_{it}-c_i}{\beta_i}$, $W_{it} = \frac{\boldsymbol{\alpha}_i'\mathbf{x}_{it}+u_{it}-c_i}{\beta_i}$, $i = 1, 2$. The disjoint regions A–E are defined by the value of W_{it} in the indicator functions. $Y^{*}_{it}(d_t) = d_t W_{it} + (1-d_t)(1+W_{it})$, that is, $Y^{*}_{it}(d_t)$ is from a mean mixture of distributions with d_t as the selection parameter, and $d_t \sim \text{Bernoulli}(\pi_d)$, where π_d is the probability of W_{it} being chosen in the mixture. A reduced form can be obtained for $N > 2$ as shown in Appendix A. However, it becomes increasingly complex as N increases and it is in general not feasible to use the reduced form for estimation.

The formulation in Region E implies that the canonical model is incoherent. While incoherence has been discussed in the literature, see for example Gourieroux, Laffont and Monfort (1980), no general treatment of incoherent system has emerged to date. It is clear that for the identification of β_i the assumption of an unbounded distribution of the $\mathbf{x}_{it}$ is sufficient. However, whether β_i can be estimated with any precision is a priori unclear and will be investigated in Monte Carlo experiments in Section 4.

A further complication for estimation is the presence of the parameters c_j, $j = 1, 2, \ldots, N$, as they are not identified under the null hypothesis $\beta_i = 0$. Inference in the presence of parameters that are not identified under the null have been studied by Andrews and Ploberger (1994) and Hansen (1996). While the consistency of β_i is not affected, the normal approximation to the distribution of the parameter is no longer valid. Hansen (1996) proposes a simulation method to assess the distribution, which does, however, exclude the presence of endogenous variables (Caner and Hansen 2004). The development of simulation methods for models with endogenous variables is beyond the scope of the present paper. I assess the impact of the uncertainty introduced by $\hat{c}_j$ on the inference by comparing the results from Monte Carlo experiments with known c_j and with estimated $\hat{c}_j$.

Finally, the contemporaneous correlation of the error term needs to be addressed. Pesaran and Pick (2007) show that integrating out the contemporaneous correlation using the common correlated effects estimator of Pesaran (2006) results in an unidentified contagion index, and this result appears to be of a general nature. Hence, below I use single equation techniques. While this foregoes efficiency gains from combining the information of different cross-section units, it allows the identification of the contagion index.

3 Instrumental variable estimation of the contagion parameter

Non-linear simultaneous equation systems have been studied extensively in the econometric literature. Rewrite Equation (1) as

$$\mathrm{f}_{it} = \mathrm{f}(\mathbf{y}_t, \mathbf{X}_t, \boldsymbol{\theta}) = u_{it},$$

where $\mathbf{y}_t$ is a $(N \times 1)$ vector of endogenous variables, $\mathbf{X}_t$ is a $(N \times K)$ matrix of exogenous regressors. Amemiya (1977) showed that in principle the optimal instrument would be $\mathrm{E}(\partial \mathrm{f}_{it}/\partial \boldsymbol{\theta}_i)$. Unfortunately, in general this expression cannot be derived and this is also true for the contagion model (1). However, Pesaran and Pick (2007) point out that the regressors in the equations for $j = 1, 2, \ldots, N$, $j \neq i$ can be used as instruments in the regression for the ith equation as they are exogenous and correlated with the endogenous variable.

Given that $\mathcal{C}_{it}$ is a non-linear transformation of the endogenous variable y_{it}, non-linear transformations of the instruments could improve the efficiency of the estimation. Kelejian (1971) suggests to approximate a non-linear function of endogenous variables by a polynomial, p_{it}, of degree M in the instruments,

$$\begin{aligned} p_{it} &= \sum_{j=1, j\neq i}^{N} (\boldsymbol{\xi}'_{1ij}\mathbf{x}_{jt} + \boldsymbol{\xi}'_{2ij}\mathbf{x}^2_{jt} + \ldots + \boldsymbol{\xi}'_{Mij}\mathbf{x}^M_{jt}) \\ &= \boldsymbol{\xi}'_i \mathbf{w}_{it}, \end{aligned} \tag{5}$$

where $\boldsymbol{\xi}_i$ is stacked vector with typical element $\boldsymbol{\xi}_{mij}$ and $\mathbf{w}_{it}$ is a stacked vector with typical element $\mathbf{x}^m_{jt}$, $m = 1, 2, \ldots, M$, and define $\mathbf{W}_i$ as the stacked (over t) counterparts of $\mathbf{w}_{it}$.

Kelejian shows that the polynomial p_{it} approximates $\mathcal{C}_{it}$ arbitrarily closely as $M \to \infty$ and that the remainder is uncorrelated with p_{it}. Newey (1990) compared the polynomial approximation with non-parametric approximations and found that the polynomial approximation leads to a lower bias and RMSE in instrumental variable estimations.

I consider two stage least square (2SLS), bias-adjusted two stage least square (B2SLS), limited information maximum likelihood (LIML), and Fuller's (1977) modified limited information maximum likelihood (LIMLk) estimators. They are special cases of the general k-class estimator, which is

$$\hat{\boldsymbol{\theta}}_i(k) = (\mathbf{Z}'_i\mathbf{P}_{\mathbf{D},i}\mathbf{Z}_i - k\mathbf{Z}'_i\mathbf{Z}_i)^{-1}(\mathbf{Z}'_i\mathbf{P}_{\mathbf{D},i}\mathbf{y}_i - k\mathbf{Z}'_i\mathbf{y}_i), \tag{6}$$

where $\mathbf{Z}_i = (\mathbf{X}_i, \boldsymbol{\mathcal{C}}_i)$, $\mathbf{D}_i = (\mathbf{X}_i, \mathbf{W}_i)$, and $\mathbf{P}_{\mathbf{D},i} = \mathbf{D}_i(\mathbf{D}'_i\mathbf{D}_i)^{-1}\mathbf{D}'_i$, $\mathbf{X}_i$ is the $(T \times K)$ matrix of predetermined regressors, $\boldsymbol{\mathcal{C}}_i$ the $(T \times 1)$ vector of contagion coefficients, $\mathbf{W}_i$ is the $(G \times T)$ matrix of instruments,

$$k = \begin{cases} 0 & \text{for 2SLS} \\ (G-K-2)/T & \text{for B2SLS} \\ k^* & \text{for LIML} \\ k^* - 1/(T-K-G) & \text{for LIML}k \end{cases}$$

and k^* is the minimum of $(\mathbf{y}_i - \boldsymbol{\theta}_i'\mathbf{Z}_i)'\mathbf{P}_{\mathbf{D},i}(\mathbf{y}_i - \boldsymbol{\theta}_i'\mathbf{Z}_i)/(\mathbf{y}_i - \boldsymbol{\theta}_i'\mathbf{Z}_i)'(\mathbf{y}_i - \boldsymbol{\theta}_i'\mathbf{Z}_i)$.

It can be shown that the consistency of the k-class estimator (6) depends on the concentration parameter (Rothenberg 1984), which is defined as

$$\mu_i = \tilde{\boldsymbol{\xi}}_i \mathbf{W}_i' \mathbf{W}_i \tilde{\boldsymbol{\xi}}_i' / \tilde{\sigma}^2_{\eta,i} \tag{7}$$

where $\tilde{\boldsymbol{\xi}}_i$ and $\tilde{\sigma}^2_{\eta,i}$ are the parameter estimate and the variance of the disturbance term, $\boldsymbol{\eta}_i$, in the first stage equation

$$\boldsymbol{\mathcal{C}}_i = \boldsymbol{\xi}_i' \mathbf{W}_i + \boldsymbol{\eta}_i.$$

The concentration parameter assumes a role similar to T in the OLS estimator and the k-class estimator will be consistent when $\mu \to \infty$.

The recent literature has investigated the behaviour of the k-class estimators under different assumption about the asymptotic behaviour of μ_i, T, and G. It has been established that for small μ_i the point estimate of β_i is biased towards the least squares estimate and that the normal approximation of the estimate is inappropriate, which can lead to incorrect inference (Phillips 1983, Nelson and Starz 1990).

3.1 Instrument selection

In the context of the contagion model with polynomial approximation a large number of instruments is available and it is not clear which of the instruments to choose. Hence, in a first stage one may wish to select the optimal set of instruments from the large pool of instruments.

A number of tests have been developed to select instruments. Donald and Newey (2001) suggest minimising the approximate MSE of k-class estimators using either cross-validation or the Mallows criterion. Alternatively, one can consider the first stage F-test or, in case of more than one endogenous right-hand side variable, the Cragg-Donald test statistic (Cragg and Donald 1993, Staiger and Stock 1997, Stock and Yogo 2005), g, which is the smallest eigenvalue of $\mathbf{G}_T$,

$$\mathbf{G}_T = \boldsymbol{\Sigma}_v^{-1/2} \mathcal{C}_i^{*\prime} \mathbf{P}_{W^*} \mathcal{C}_i^* \boldsymbol{\Sigma}_v^{-1/2} / G, \tag{8}$$

where for any matrix Π, $\Pi^* = \mathbf{M}_{X,i}\Pi$, $\mathbf{M}_{X,i} = (\mathbf{I} - \mathbf{X}_i(\mathbf{X}_i'\mathbf{X}_i)^{-1}\mathbf{X}_i')$, and $\boldsymbol{\Sigma}_v = \mathcal{C}_i^{*\prime}(\mathbf{I} - \mathbf{P}_{D,i})\mathcal{C}_i^*/(T-K-G)$. It is easy to establish that g is the F-test statistic when $G = 1$.

The set of instruments are chosen that maximise the F-statistic or the Cragg-Donald test statistic. Alternatively, using critical values tabulated by Stock and Yogo (2005) the power of the instruments can be assessed with respect the bias of the parameter estimate or the coverage ratio of the confidence interval.

However, Chao and Swanson (2005) argue that selecting instruments may lead to a lower concentration parameter and therefore to inferior inference. In particular, when strong instruments are not available using all instruments may be a valid strategy.

3.2 μ-similar tests

Given that the properties of the k-class estimators depend on the concentration parameter, μ, tests that are similar with respect to μ would be desirable. In contrast to k-class estimators such μ-similar tests have the same size independent of μ, that is, independent of the strength of the instruments. This can be achieved either by finding a statistic that does not depend on the nuisance parameter, μ, or by conditioning the critical values on μ so that the test will have the correct size independent of its value (Andrews and Stock 2006).

The μ-similar tests are the Anderson and Rubin (1949) test, the LM test of Moreira (2001) and Kleibergen (2002), and the conditional likelihood ratio test (CLR) of Moreira (2003). Andrews and Stock (2006) compare the power of the three μ-similar tests in a many weak instruments asymptotics framework. They find that the CLR test is on the asymptotic power envelope.

The tests are defined as follows. Define the vectors $\mathbf{a}_0 = [\beta_i, 1]'$ and $\mathbf{b}_0 = [1, -\beta_i]'$, and

$$\mathbf{S}_i = \frac{(\bar{\mathbf{W}}_i'\bar{\mathbf{W}}_i)^{-1/2}\bar{\mathbf{W}}_i'\bar{\boldsymbol{\mathcal{C}}}_i\mathbf{b}_0}{(\mathbf{b}_0'\hat{\boldsymbol{\Omega}}\mathbf{b}_0)^{1/2}},$$

$$\mathbf{V}_i = \frac{(\bar{\mathbf{W}}_i'\bar{\mathbf{W}}_i)^{-1/2}\bar{\mathbf{W}}_i'\bar{\boldsymbol{\mathcal{C}}}_i\mathbf{a}_0}{(\mathbf{a}_0'\hat{\boldsymbol{\Omega}}\mathbf{a}_0)^{1/2}},$$

where $\hat{\boldsymbol{\Omega}} = \bar{\boldsymbol{\mathcal{C}}}_i\mathbf{M_D}\bar{\boldsymbol{\mathcal{C}}}_i/(T-K-G)$ and $\bar{\mathbf{W}}_i = \mathbf{M_X}\mathbf{W}_i$, $\bar{\boldsymbol{\mathcal{C}}}_i = \mathbf{M_X}\boldsymbol{\mathcal{C}}_i$, with $\mathbf{M_X} = \mathbf{I} - \mathbf{X}_i(\mathbf{X}_i'\mathbf{X}_i)^{-1}\mathbf{X}_i'$, and $\mathbf{M_D}$ defined analogously.

Then the Anderson and Rubin (1949) test is defined as

$$\mathrm{AR}(\beta_i) = \frac{\mathbf{S}_i'\mathbf{S}_i}{G},$$

which is asymptotically distributed as χ^2_G/G under the null. The LM-test is defined as

$$\mathrm{LM}(\beta_i) = \frac{(\mathbf{S}_i'\mathbf{V}_i)^2}{\mathbf{V}_i'\mathbf{V}_i},$$

which is asymptotically distributed as χ_1^2 under the null. The CLR test is

$$\text{CLR}(\beta_i) = \frac{1}{2}\left(\mathbf{S}_i'\mathbf{S}_i - \mathbf{V}_i'\mathbf{V}_i + \sqrt{(\mathbf{S}_i'\mathbf{S}_i + \mathbf{V}_i'\mathbf{V}_i)^2 - 4[(\mathbf{S}_i'\mathbf{S}_i)(\mathbf{V}_i'\mathbf{V}_i) - (\mathbf{S}_i'\mathbf{V}_i)^2]}\right),$$

which has no closed form distribution. However, conditional on G and $\mathbf{V}_i'\mathbf{V}_i$ the distribution can be computed by Monte Carlo simulations.

I will refer to them as "μ-similar" tests instead of "similar" tests as in the literature because the presence of the unknown threshold parameters, c_i, means that they are no longer similar when applied to the contagion model. The Monte Carlo experiments below will investigate how far the estimators are affected by the threshold parameter.

4 Monte Carlo evidence

4.1 Experimental design

The Monte Carlo experiments are based on artificial data with $N = 3$ and $T = 300$. The advantage of small N is that the data with $\beta \neq 0$ can be generated from the reduced form reported in Appendix A. It might seem appealing to use the reduced form for the estimation of the system. This, however, would reduce the scope of the estimation method to cases with $N = 2$ and $N = 3$ as the reduced form becomes highly complex as N increases. In order to obtain generally valid results, I use the instrumental variable techniques discussed above.

The structural equation is

$$y_{it}^{(r)} = \alpha_i x_{it}^{(r)} + \beta_i \mathcal{C}_{it}^{(r)} + u_{it}^{(r)}. \tag{9}$$

The exogenous variable is generated as $x_{it}^{(r)} \sim \text{N}(0, 1)$. The error term is generated using the following standardised one-factor structure

$$u_{it}^{(r)} = \frac{\gamma_i^{(r)} f_t^{(r)} + \varepsilon_{it}^{(r)}}{\sqrt{1 + \gamma_i^{(r)2}}}$$

where $\gamma_i^{(r)} \sim \text{U}\left(\frac{1}{2}\gamma, \frac{3}{2}\gamma\right)$ is a scalar, $\text{U}(a, b)$ denotes the uniform distribution with lower bound a and upper bound b, $f_t^{(r)} \sim \text{iid}\,\text{N}(0, 1)$, and $\varepsilon_{it}^{(r)} \sim \text{iid}\,\text{N}(0, 1)$, $r = 1, 2, \ldots, R$, and R denotes the number of replications, which is set to $R = 500$ in the experiments reported below. Under these assumptions the error term, $u_{it}^{(r)}$, has expected value of zero and a unit variance. The pairwise correlation coefficient of the errors is given by

$$\text{Corr}\left(u_{it}^{(r)}, u_{jt}^{(r)}\right) = \frac{\gamma_i^{(r)}\gamma_j^{(r)}}{\sqrt{(1 + \gamma_i^{(r)2})(1 + \gamma_j^{(r)2})}}.$$

In any application it is likely that contemporaneous correlation will be present, and in the simulations γ is set to 1.

As discussed above, attempts to filter out the contemporaneous correlation have the problem that contagion becomes unidentified. Hence, I use single equation estimation methods and apply them to the first cross section unit, $i = 1$.

Given that $\mathrm{Var}\left(u_{it}^{(r)}\right) = \mathrm{Var}\left(x_{it}^{(r)}\right) = 1$, the correlation between $y_{it}^{(r)}$ and $x_{it}^{(r)}$ is $\alpha_i/\sqrt{\alpha_i^2+1}$, that is, an increasing function in α_i. As $\mathcal{C}_{it}^{(r)}$ is an increasing function in $y_{it}^{(r)}$, this implies that the instruments, i. e. the $N-1$ regressors $x_{jt}^{(r)}$, $j = 1, 2, \dots, N, j \neq i$, become stronger as the α_j increase. I set $\alpha_i = 0.1$, $\forall i$, to simulate estimation with weak instruments and $\alpha_i = 1$, $\forall i$, to simulate estimation with stronger instruments.

In order to evaluate the effect of the incoherence of the system on the estimations and the tests for contagion I use two specifications. In the first specification, $\pi_d = 0$, where π_d is the probability of a crisis in region E in (4). Hence, a crisis will always occur in the indeterminate region and the system is coherent. In the second specification, $\pi_d = 0.5$, that is the probability of a crisis in the indeterminate region is 0.5, which is independent of the right hand side variables and the system is therefore incoherent.

In the data generating process I set $c_i = c = 1.64$, $\forall i$. This choice implies that in the Monte Carlo experiment it is unlikely that the contagion index, $\mathcal{C}_{it}$, becomes a vector of ones or zeros, which happens if the proportion of crises periods is too high or too low. In neither case would the contagion index be identified.

In the first set of Monte Carlo experiments it is assumed that c is known. This simplifies inference, and gives a useful benchmark to compare the results when this assumption is dropped. The contagion coefficient is estimated using OLS, 2SLS, B2SLS, LIML, and LIMLk. The instruments are a power series of the regressors of the cross section units $i = 2, 3$, where the maximum power is $M = 6$. Additionally, optimal subsets of powers are selected using the minimum approximate MSE of the IV estimators based on cross validation criterion and Mallow's criterion (Donald and Newey 2001), and the first stage F-statistic (Stock and Yogo 2005).

Furthermore, inference is made using the Anderson-Rubin (1949) test, the LM-test of Moreira (2001) and Kleibergen (2002), and the CLR test of Moreira (2003) described in Section 3.2. The instruments for the μ-similar tests are again the power series of regressors up to power $M = 6$ and the optimal subset of powers selected by the first-stage F-statistic.

In a second set of Monte Carlo experiments c is assumed to be unknown and estimated using a grid search over the grid $\{1.4, 1.52, 1.56, 1.6, 1.64, 1.68, 1.72, 1.76, 1.88\}$. The grid ensures that crises remain a tail event and that a sufficient number of observations is available for estimation. The value of $\hat{c}$ is estimated by minimising the squared errors estimations using all instruments for each estimator. For the μ-similar tests, the $\hat{c}$ obtained from GIVE is used with all instruments. Subject to the estimated $\hat{c}$, the instrumental variable estimators and robust tests are used as above.

Reported are the median bias, median$(\hat{\beta}_i^{(r)} - \beta_i)$, the median absolute bias (MAB), median(abs$(\hat{\beta}_i^{(r)} - \beta_i)$), and the rejection probability of the null hypothesis $H_0 : \beta_i = 0$, $\frac{1}{R}\sum_{r=1}^{R}(\text{abs}(t^{(r)}) > 1.64)$, where t denotes the t-statistic of the contagion parameter obtained from the respective estimator.

4.2 Results

Table 1 reports the median bias and the MAB and Table 2 the size and power for OLS and 2SLS, B2SLS, LIML, LIMLk using different instrument selection methods and different parameter combinations and under the assumption that c is known. The first three columns give the results for $\alpha = 0.1$, i. e. weak instruments, and the fourth to sixth column the results for $\alpha = 1$, i. e. stronger instruments. When $\alpha = 0.1$ the F-statistics for the instrument set selected based on the maximum F-statistic are between 2 and 3 for the different parameter constellations. When $\alpha = 1$ the F-statistic is greater than 20.

In the first column are the results for weak instruments and no contagion. OLS has a large bias and MAB, and the null of no contagion is rejected with near certainty. The median bias of instrumental variable estimators is also considerable, which confirms the well known results from the literature, e. g. Nelson and Starz (1990). When the instruments are selected by maximising the first state F-test statistic, the bias of 2SLS and B2SLS are even larger than that of OLS. The smallest bias is achieved using all instruments and LIML or LIMLk.

The MAB for many of the instrumental variables techniques is larger than that of OLS. Only the MAB of 2SLS using the instruments that minimise the MSE based on Mallow's criterion and that of LIML based on the MSE using cross-validation are smaller than that of OLS. The small bias of using LIML or LIMLk with all instruments is bought at the expense of a large MAB. Using 2SLS and B2SLS with all instruments may be a reasonable compromise as their bias is considerably smaller than that of OLS but there MAB only somewhat larger.

The size of the test for contagion based on the instrumental variable methods is much better than that of OLS, yet quite far from the nominal size of 5%. 2SLS and B2SLS with all instruments have the best size, which, however, is still about twice the nominal size.

In the presence of contagion the bias and MAB of OLS increase and the power is 1. The conclusions from the case of $\beta_i = 0$ for the instrumental variable techniques carries over. LIML and LIMLk with all instruments have the smallest bias but considerable MAB and 2SLS and B2SLS with all

Table 1: Median bias and median absolute bias in Monte Carlo, given c

		$\alpha = 0.1$			$\alpha = 1$		
		$\beta = 0$	$\beta = 1$	$\beta = 1$	$\beta = 0$	$\beta = 1$	$\beta = 1$
			$\pi_d = 0$	$\pi_d = 0.5$		$\pi_d = 0$	$\pi_d = 0.5$
				Median bias			
OLS		1.034	1.147	1.124	0.707	0.865	0.829
2SLS	CV	0.876	1.019	1.054	0.167	0.069	0.097
	Mallow	0.910	1.072	1.082	0.220	0.098	0.126
	F	1.256	1.034	1.087	0.129	−0.030	0.006
	all	0.633	0.501	0.515	0.142	−0.016	0.012
B2SLS	CV	0.934	1.030	1.095	0.204	0.064	0.084
	Mallow	0.786	0.986	1.035	0.119	0.014	0.045
	F	1.256	1.034	1.088	0.129	−0.030	0.006
	all	0.575	0.502	0.586	0.111	−0.031	−0.017
LIML	CV	0.858	0.992	1.044	0.135	0.000	0.024
	Mallow	0.882	0.962	0.946	0.150	0.013	0.035
	F	0.776	0.921	0.990	0.115	0.001	0.027
	all	0.410	0.306	0.418	0.111	−0.063	−0.028
LIMLk	CV	0.910	1.072	1.082	0.220	0.098	0.126
	Mallow	0.729	0.972	1.040	0.131	0.000	0.024
	F	0.835	0.905	0.915	0.143	−0.002	0.024
	all	0.410	0.306	0.418	0.111	−0.063	−0.028
				Median absolute bias			
OLS		1.034	1.147	1.124	0.707	0.865	0.829
2SLS	CV	1.166	1.023	1.135	0.306	0.189	0.182
	Mallow	1.025	1.073	1.100	0.320	0.191	0.185
	F	2.205	1.714	1.963	0.303	0.200	0.188
	all	1.246	0.794	0.900	0.295	0.187	0.179
B2SLS	CV	1.195	1.057	1.139	0.321	0.191	0.177
	Mallow	1.520	1.042	1.193	0.312	0.183	0.172
	F	2.205	1.714	1.963	0.303	0.200	0.188
	all	1.240	0.796	0.922	0.302	0.205	0.191
LIML	CV	1.184	1.023	1.142	0.316	0.204	0.184
	Mallow	1.366	1.045	1.140	0.305	0.183	0.173
	F	1.861	1.169	1.378	0.304	0.194	0.187
	all	2.111	1.279	1.465	0.290	0.203	0.191
LIMLk	CV	1.025	1.073	1.100	0.320	0.191	0.185
	Mallow	1.352	1.042	1.222	0.315	0.204	0.184
	F	1.719	1.138	1.261	0.299	0.191	0.185
	all	2.111	1.279	1.465	0.290	0.203	0.191

CV denotes the estimates obtained using the instruments selected using the minimum MSE based on the cross-validation criterion, Mallow denotes those obtained using the instruments selected using the minimum MSE based on Mallow's criterion, F those obtained using the instruments selected based on the first stage F-statistic, and all those using all instruments.

Table 2: Size and power in Monte Carlo, given c

		$\alpha = 0.1$			$\alpha = 1$		
		$\beta = 0$	$\beta = 1$	$\beta = 1$	$\beta = 0$	$\beta = 1$	$\beta = 1$
			$\pi_d = 0$	$\pi_d = 0.5$		$\pi_d = 0$	$\pi_d = 0.5$
OLS		0.992	1.000	1.000	0.946	1.000	1.000
2SLS	CV	0.164	0.746	0.684	0.120	0.970	0.978
	Mallow	0.260	0.928	0.878	0.140	0.982	0.988
	F	0.172	0.484	0.462	0.128	0.880	0.922
	all	0.102	0.410	0.352	0.110	0.916	0.944
B2SLS	CV	0.190	0.794	0.730	0.142	0.970	0.976
	Mallow	0.116	0.586	0.520	0.104	0.938	0.952
	F	0.172	0.484	0.462	0.128	0.880	0.922
	all	0.082	0.456	0.400	0.086	0.868	0.908
LIML	CV	0.144	0.716	0.630	0.098	0.890	0.920
	Mallow	0.138	0.618	0.534	0.112	0.940	0.954
	F	0.108	0.494	0.458	0.106	0.926	0.948
	all	0.148	0.396	0.354	0.126	0.860	0.910
LIMLk	CV	0.260	0.928	0.878	0.140	0.982	0.988
	Mallow	0.124	0.640	0.570	0.098	0.892	0.920
	F	0.130	0.526	0.472	0.116	0.924	0.948
	all	0.148	0.396	0.354	0.126	0.860	0.910

See footnote of Table 1.

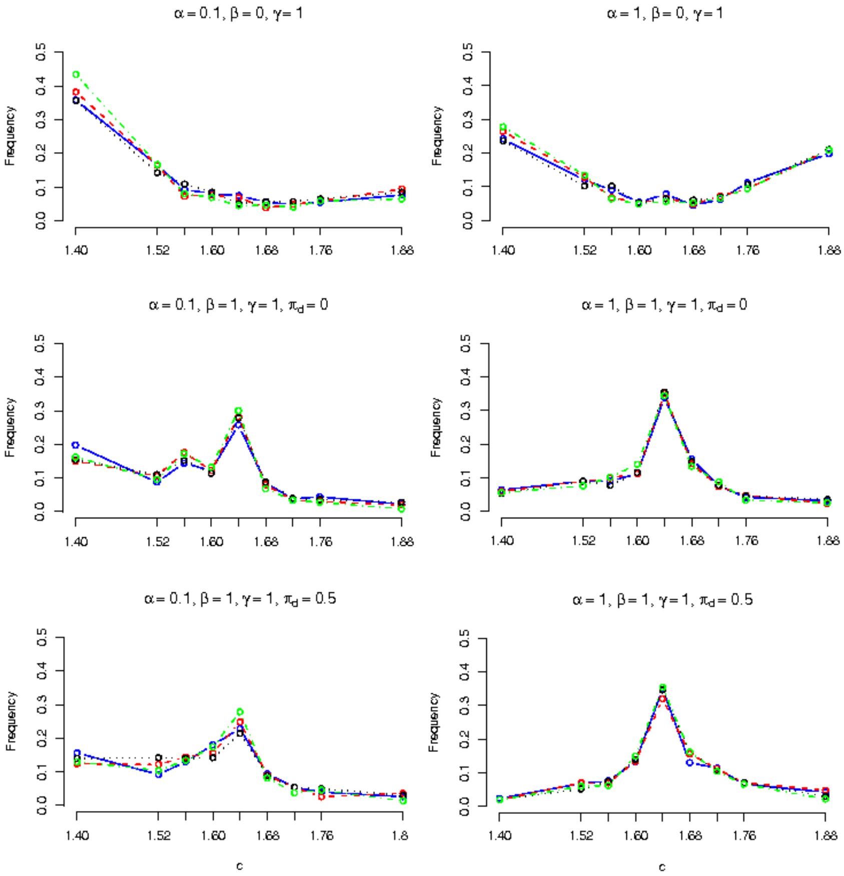
instruments have a small bias and MAB. The power of the tests with the best size is about 0.4-0.5.

When comparing the results for the coherent system with those of the incoherent system, it can be observed that nearly uniformly the bias and the MAB is larger and the power lower in the incoherent system. However, the differences are relatively small.

The right hand side of Tables 1 and 2 suggest that the bias and MAB are substantially reduced as the strength of the instruments increases. In the presence of contagion, the parameter estimates are essentially unbiased. The bias and MAB for the instrumental variable estimation are much smaller that that of OLS. While closer to the nominal size, the tests still have size of around 10% instead of the nominal 5%. The power of the tests increases to around 90%.

In the above experiments, it was assumed that the threshold level for a crisis, c, was known. This assumption is now dropped and c is estimated by grid search. Figure 1 shows the frequencies of the estimated $\hat{c}$ for each point in the grid for each of the instrument selection criteria and for all instruments in the case of 2SLS. The two top graphs show the frequency of $\hat{c}$ under a

Figure 1: Estimation of $\hat{c}$ in the Monte Carlo experiments using 2SLS



The probabilities are $\frac{1}{R}\sum_{r=1}^{R}\mathrm{I}(\hat{c}^{(r)} = c_i)$, where $c_i \in \{1.4, 1.52, 1.56, 1.6, 1.64, 1.68, 1.72, 1.76, 1.88\}$, $\hat{c}^{(r)}$ was estimated using grid search over c_i minimising the squared errors of 2SLS estimations: the blue, solid line is for 2SLS-CV, the red, dashed line for 2SLS-Mallow, the black, dotted line for 2SLS-F, and the green, dash-dotted line for 2SLS-all.

data generating process (DGP) with $\beta = 0$. In this case c is not identified, and as a consequence the estimates are fairly evenly distributed with the extreme values somewhat more often selected. The other four graphs show the estimated $\hat{c}$ under a DGP with $\beta = 1$, in which case c is identified, and for all instrument selection criteria the $\hat{c}$ that is estimated most frequently is now the correct value of 1.64. It can also be seen that the estimation of $\hat{c}$ is relatively similar for the instrument selection criteria, although using all instruments yields marginally better estimates for three out of the four cases with $\beta_i = 1$.

The median bias, MAB, and the size and power with estimated $\hat{c}$ are reported in Tables 3 and 4. The theoretical results from the literature mentioned above suggest that the point estimates remains consistent but that the normal approximation of the parameter distribution is no longer valid. The results reflect this.

The bias under weak instruments is only slightly larger in most cases. The exception is the bias for the estimators that use all instruments, which no longer have a smaller bias than the estimations with subset of the instruments but are of similar magnitude. Interestingly, the MAB is generally smaller in the case of estimated c. When the instruments are strong, the estimates are no longer unbiased even if the bias is small it is now clearly positive for all estimators and instrument selection methods.

A further interesting result is that the size of the test under weak instruments is close to the nominal size. This does, however, not suggest that the test is now the right one but that the effect of the weak instruments is in this case counter-acted by deviation from normality. That the test is not correct is demonstrated by the case of strong instruments, where the results are similar to the case where c is assumed to be known.

Table 5 reports the results for the three similar tests. The simulation results in the top panel assume that c is known. The instruments are selected either using the first-stage F-statistic or all instruments are used. The size of the test is approximately the correct 5% under all scenarios. The power of the tests, however, depends crucially on the strength of the instruments. Under weak instruments, the power of the tests is low, while under strong instruments the test has good power.

Comparing the instrument selection methods, it appears that the power of the tests that use the instruments based on the first-stage F-statistic is slightly higher than that of tests that use all instruments. When comparing the three similar tests it is apparent that the CLR test and the LM test have very similar power, whereas the AR test has relatively lower power. Incoherence of the model reduces the power of all three tests only slightly.

The lower half of the table reports the results with an estimated threshold. Compared to the results of the tests with known c, the size and power of the tests decreases when instruments are weak. When the instruments are strong the test is still undersized whereas the power is not affected.

5 Contagion between stock markets

The techniques discussed are now used to test for contagion between the stock markets in the USA, the UK, and Japan. A first consideration is the frequency of data to be analysed. Ideally, the frequency would be as high as possible so that crises observations are not mixed with non-crises observations. However, it has been established in the literature that financial variables are very difficult to forecast over short horizons. In order to have instruments with some power forecastability is important and I will therefore

Table 3: Median bias and median absolute bias in Monte Carlo, estimated $\hat{c}$

		$\alpha = 0.1$			$\alpha = 1$		
		$\beta = 0$	$\beta = 1$ $\pi_d = 0$	$\beta = 1$ $\pi_d = 0.5$	$\beta = 0$	$\beta = 1$ $\pi_d = 0$	$\beta = 1$ $\pi_d = 0.5$
			Median bias				
OLS		1.012	1.137	1.136	0.783	0.869	0.864
2SLS	CV	0.912	1.069	1.036	0.211	0.117	0.122
	Mallow	0.919	1.088	1.118	0.250	0.155	0.159
	F	1.013	1.193	1.300	0.143	0.005	0.030
	all	0.842	1.037	1.044	0.169	0.051	0.074
B2SLS	CV	0.939	1.060	1.076	0.325	0.195	0.214
	Mallow	1.067	1.079	1.116	0.308	0.192	0.204
	F	1.405	1.576	1.577	0.296	0.197	0.209
	all	0.966	1.086	1.076	0.355	0.192	0.230
LIML	CV	0.921	1.078	1.067	0.207	0.038	0.075
	Mallow	0.856	1.032	1.033	0.176	0.047	0.074
	F	0.693	1.034	1.011	0.142	0.047	0.063
	all	0.706	1.029	1.054	0.118	0.012	0.040
LIMLk	CV	0.919	1.088	1.118	0.250	0.155	0.159
	Mallow	0.892	1.077	1.051	0.201	0.038	0.075
	F	0.799	1.005	1.034	0.159	0.038	0.064
	all	0.706	1.029	1.054	0.118	0.012	0.040
			Median absolute bias				
OLS		1.012	1.137	1.136	0.783	0.869	0.864
2SLS	CV	0.953	1.072	1.045	0.322	0.195	0.198
	Mallow	0.922	1.088	1.121	0.316	0.208	0.213
	F	1.405	1.576	1.577	0.296	0.197	0.209
	all	0.988	1.051	1.085	0.294	0.188	0.203
B2SLS	CV	0.939	1.060	1.076	0.325	0.195	0.214
	Mallow	1.067	1.079	1.116	0.308	0.192	0.204
	F	1.405	1.576	1.577	0.296	0.197	0.209
	all	0.966	1.086	1.076	0.355	0.192	0.230
LIML	CV	0.933	1.078	1.074	0.361	0.196	0.230
	Mallow	1.041	1.061	1.099	0.305	0.197	0.208
	F	1.311	1.197	1.227	0.307	0.186	0.197
	all	1.234	1.211	1.286	0.281	0.202	0.198
LIMLk	CV	0.922	1.088	1.121	0.316	0.208	0.213
	Mallow	0.975	1.100	1.091	0.362	0.196	0.230
	F	1.224	1.185	1.205	0.299	0.187	0.198
	all	1.234	1.211	1.286	0.281	0.202	0.198

The threshold c is estimated by minimising the squared errors of 2SLS using all instruments. For the notation see footnote to Table 1.

Table 4: Size and power in Monte Carlo, estimated $\hat{c}$

		$\alpha = 0.1$			$\alpha = 1$		
		$\beta = 0$	$\beta = 1$ $\pi_d = 0$	$\beta = 1$ $\pi_d = 0.5$	$\beta = 0$	$\beta = 1$ $\pi_d = 0$	$\beta = 1$ $\pi_d = 0.5$
OLS		1.000	1.000	1.000	0.986	1.000	1.000
2SLS	CV	0.118	0.828	0.760	0.124	0.976	0.978
	Mallow	0.194	0.954	0.930	0.136	0.990	0.982
	F	0.070	0.510	0.526	0.108	0.910	0.928
	all	0.064	0.566	0.500	0.118	0.944	0.946
B2SLS	CV	0.124	0.862	0.798	0.138	0.976	0.974
	Mallow	0.082	0.644	0.536	0.108	0.946	0.956
	F	0.070	0.510	0.526	0.108	0.910	0.928
	all	0.044	0.608	0.546	0.108	0.912	0.896
LIML	CV	0.058	0.792	0.720	0.114	0.924	0.906
	Mallow	0.074	0.648	0.576	0.122	0.950	0.956
	F	0.078	0.542	0.480	0.102	0.948	0.950
	all	0.080	0.418	0.414	0.112	0.916	0.904
LIMLk	CV	0.194	0.954	0.930	0.136	0.990	0.982
	Mallow	0.048	0.704	0.622	0.114	0.924	0.908
	F	0.064	0.566	0.500	0.118	0.944	0.946
	all	0.080	0.418	0.414	0.112	0.916	0.904

See footnote of Table 3.

use quarterly stock market returns.

More precisely, the data are quarterly growth rates of the stock market indices in the UK, the USA, and Japan on a monthly frequency for the period between March 1971 and May 2006 as reported in the OECD Main Economic Indicators. I use the growth rate of industrial production, the growth rate of the short term interest rate, and the growth rate of the price of crude oil as explanatory variables. These variables are lagged by four month to ensure exogeneity. Details of the data are in Appendix B.

The regression equation is

$$\Delta s_{it} = \alpha_{0i} + \delta_i \Delta oil_{t-4} + \alpha_{1i} \Delta ip_{i,t-4} + \alpha_{2i} \Delta r_{i,t-4} + \beta_i \mathcal{C}_{it} + \varepsilon_{it} \tag{10}$$

where Δ denotes growth rates, s_{it} is the stock market index, oil_{it} is the price of crude oil, ip_{it} is the index of industrial production, and r_{it} is the short term interest rate. The contagion index, $\mathcal{C}_{it}$, is defined in (2).

In the instrumental variable estimations the instruments are

$$\mathbf{W}_{it} = (\Delta ip_{j,t-4}, \Delta r_{j,t-4}, \Delta ip_{l,t-4}, \Delta r_{l,t-4}),$$

Table 5: Rejection probability of μ-similar tests in Monte Carlo

		$\alpha = 0.1$			$\alpha = 1$		
		$\beta = 0$	$\beta = 1$	$\beta = 1$	$\beta = 0$	$\beta = 1$	$\beta = 1$
			$\pi_d = 0$	$\pi_d = 0.5$		$\pi_d = 0$	$\pi_d = 0.5$
				given c			
CLR	F	0.064	0.120	0.110	0.048	0.766	0.772
	all	0.062	0.064	0.054	0.052	0.720	0.694
LM	F	0.066	0.130	0.114	0.048	0.752	0.766
	all	0.064	0.074	0.062	0.052	0.712	0.688
AR	F	0.048	0.102	0.104	0.044	0.598	0.588
	all	0.050	0.046	0.058	0.042	0.376	0.334
				estimated $\hat{c}$			
CLR	F	0.024	0.098	0.098	0.048	0.756	0.778
	all	0.040	0.034	0.056	0.028	0.768	0.734
LM	F	0.024	0.108	0.104	0.046	0.746	0.730
	all	0.050	0.046	0.054	0.028	0.762	0.730
AR	F	0.034	0.074	0.080	0.040	0.602	0.588
	all	0.052	0.056	0.072	0.040	0.368	0.328

CLR denotes the conditional likelihood ratio test (Moreira 2003), LM the Lagrange multiplier test of Moreira (2001) and Kleibergen (2002), and AR the Anderson-Rubin (1949) test. F indicates that the instruments were selected by the first stage F-test, all that all instruments were used.

where $l, j = \{\text{UK}, \text{USA}, \text{Japan}\}$, $j \neq l$, $j, l \neq i$, and powers of $\mathbf{W}_{it}$. The growth rate of the oil price is common in the regressions for the three stock markets and can therefore not function as an instrument for the contagion index.

The crisis thresholds c_{ij} and c_{lt} were allowed to vary between the countries,

$$\mathcal{C}_{it} = \mathrm{I}(\mathrm{I}(y_{jt} - c_{ij}) + \mathrm{I}(y_{lt} - c_{il}))\,.$$

for each i, j, and l. The thresholds were estimated by minimising the squared errors from 2SLS estimations using all instruments, which performed slightly better than the selection methods in the Monte Carlo experiments. The grid was chosen such that for each country between 2.5% and 10% of observations are crisis observations and the grid points were set such that all possible combination of crises were covered. Alternatively, the thresholds could be estimated using either one of the instrumental selection methods or a different estimator. The results from using these methods are very similar to the

Table 6: Threshold values

	$c_{i,UK}$	$c_{i,USA}$	$c_{i,Japan}$
UK		-6.6	-10.4
USA	-11.0		-10.6
Japan	-14.0	-6.8	

Each row gives the thresholds for the regression for the respective stock-market.

one from 2SLS with all instruments and are therefore not reported here.

Table 6 reports the estimated thresholds. The first row gives the thresholds for the US and the Japanese stock markets in the equation for the UK stock market. Assuming that the contagion index enters the regression equation significantly, contagion occurs if the US stock markets falls by at least 6.6% or if the Japanese stock market falls by at least 10.4%. The thresholds for the US and Japanese stock markets are similar in each of the equations they enter. In contrast, the value of the UK stock market is smaller for the regression for the US stock market than for the Japanese stock market, which would imply that a larger fall in the UK stock market is necessary to cause contagion to the Japanese stock market than to that in the USA. This could be interpreted as a larger financial distance between the UK and Japan than the UK and the USA. However, these interpretations are subject to the finding of a significant contagion coefficient.

Conditional on the thresholds the contagion coefficient was estimated using OLS, 2SLS, B2SLS, LIML, and LIMLk, with the instruments selected by the different instrument selection methods. Table 7 reports the results for the three countries. The OLS estimates are all highly significant with absolute t-values of 8.909, 6.904, and 8.168, which as discussed above can be the result of actual contagion as well as contemporaneous correlation.

For the UK stock market, the point estimates vary dramatically between the estimation methods, which is likely to be a result of the weakness of the instruments with a maximum first-stage F-test statistic of 3.610. The parameters are significant for a subset of the estimation methods. However, as the Monte Carlo experiments showed that the tests are oversized this finding is therefore not conclusive.

The maximum first-stage F-test statistic is 6.599 for the US stock market, which is still well below the level of about 20 achieved in the Monte Carlo experiments with $\alpha_i = 1$. The point estimates are however all negative and, except for those where the instruments were selected by the F-test statistics and B2SLS with all instruments, significant.

The parameters for the Japanese stock market are also all negative but

Table 7: Results for estimation using stock market data

	CV	Mallow	F	all
	UK ($F = 3.610$)			
OLS	−11.374			
	(8.909)			
2SLS	0.362	0.362	3.571	−14.784
	(0.723)	(0.723)	(0.436)	(2.178)
B2SLS	−11.703	4.050	6.017	−19.108
	(0.894)	(0.117)	(0.659)	(6.356)
LIML	−11.151	−11.151	25.563	−12.414
	(4.270)	(4.270)	(1.359)	(3.703)
LIMLk	−11.153	−11.153	20.293	−12.406
	(4.291)	(4.291)	(1.279)	(3.731)
	USA ($F = 6.599$)			
OLS	−8.552			
	(6.904)			
2SLS	−7.852	−7.852	−4.003	−9.688
	(3.027)	(3.027)	(1.011)	(2.310)
B2SLS	−7.617	−7.617	−3.612	−3.483
	(2.556)	(2.556)	(0.877)	(0.649)
LIML	−7.295	−7.295	−3.735	−8.472
	(2.174)	(2.174)	(0.921)	(4.387)
LIMLk	−7.330	−7.330	−3.929	−8.472
	(2.213)	(2.213)	(0.991)	(4.409)
	Japan ($F = 7.490$)			
OLS	−10.278			
	(4.850)			
2SLS	−6.183	−6.183	−6.183	−10.769
	(1.083)	(1.083)	(1.083)	(2.191)
B2SLS	−10.802	−5.957	−5.957	−11.166
	(1.982)	(1.014)	(1.014)	(1.991)
LIML	−11.492	−5.715	−5.715	−11.492
	(1.919)	(0.946)	(0.946)	(1.919)
LIMLk	−11.426	−5.846	−5.846	−11.426
	(1.938)	(0.984)	(0.984)	(1.938)

The threshold value, $\hat{c}$, was estimated by minimising the squared errors of the regression equation with parameters estimated by 2SLS using all instruments. For notation see the footnote of Table 1.

Table 8: Results for similar tests using stock market data

	F			all		
	UK	USA	Japan	UK	USA	Japan
CLR	1.601	0.700	2.671	0.154	2.627	1.488
	(3.930)	(3.710)	(3.710)	(3.845)	(3.710)	(3.545)
LM	1.577	0.700	2.669	0.150	2.606	1.479
	(3.841)	(3.841)	(3.841)	(3.841)	(3.841)	(3.841)
AR	3.015	0.481	1.249	3.553	1.521	1.675
	(2.372)	(2.372)	(2.372)	(1.517)	(1.517)	(1.517)

The table reports the statistic for the stock market data with the critical values in brackets in the row below. $\hat{c}$ was estimated by minimising the squared errors of Equation (1) using 2SLS using all powers. For notation see the footnote of Table 5.

only significant for 2SLS with all instruments and B2SLS for the instruments selected minimising the MSE using cross-validation and all instruments. While still small in absolute terms, the first-stage F-test statistic is the highest among the three stock markets at 7.490.

Table 8 reports the results of the similar tests. The table gives the results from using the instruments selected with the first stage F-test on the left hand side and the results obtained using all instruments on the right hand side. For each test the statistic is reported in the top line and the critical value in brackets below.

The CLR-test and the LM-test cannot reject the null of no contagion for all markets and instrument selection methods. The AR-test rejects the null of no contagion for the UK stock market using the instruments suggested by the first stage F-test, and for all markets using all instruments. However, given that in the Monte Carlo experiments the size and power properties of the tests have been demonstrated to be poor, the results cannot be given much weight.

Based on the evidence from the instrumental variables test the case for contagion is a weak one. However, this may as much be due to the weakness of the instruments, and therefore implicitly due to the well known difficulty to forecast the stock market even over longer horizons.

6 Conclusion

This paper considers the estimation of a contagion coefficient, where the contagion index in the canonical model of Pesaran and Pick (2007) is endogenous. A number of issues arise that need to be addressed. The first issue is the quality of the estimates obtained from instrumental variable estimators under weak instruments. Monte Carlo experiments suggest that weak instruments can lead to serious bias and incorrect inference when standard instrumental variable techniques are used. New robust tests do, however, have the correct size even if instruments are weak.

Second, the canonical model of contagion is incoherent. Finally, the canonical model of contagion contains a parameter that is only identified under the alternative hypothesis, which has implications for the distribution of the contagion parameter. Monte Carlo experiments indicate that standard tests are not overly affected by these issues and that the strength of the instruments remains the key to valid inference.

Contagion was then tested using stock market data from the UK, the USA, and Japan. Tests based on OLS, i. e. without correction for the endogeneity of the contagion index, are highly significant, which mirrors the results in the empirical literature on financial contagion. Using the instrumental variable methods the results are mixed, which may largely be attributed to the weakness of the instruments. However, financial variables are known to be difficult to forecast and strong instruments are therefore difficult to find. Future research will hopefully fill this gap and supply instruments that can successfully be used in tests of contagion.

A Appendix: Reduced form for $N > 2$

When $N = 3$ and using definition (2) of the contagion index, the system of equations is

$$\begin{aligned}
y_{1t} &= \boldsymbol{\alpha}_1'\mathbf{x}_{1t} + \beta_1 \mathrm{I}(\mathrm{I}(y_{2t} > c_2) + \mathrm{I}(y_{3t} > c_3)) + u_{1t}, \\
y_{2t} &= \boldsymbol{\alpha}_2'\mathbf{x}_{2t} + \beta_2 \mathrm{I}(\mathrm{I}(y_{1t} > c_1) + \mathrm{I}(y_{3t} > c_3)) + u_{2t}, \\
y_{3t} &= \boldsymbol{\alpha}_3'\mathbf{x}_{3t} + \beta_3 \mathrm{I}(\mathrm{I}(y_{1t} > c_1) + \mathrm{I}(y_{2t} > c_2)) + u_{3t},
\end{aligned}$$

which can be written equivalently as

$$Y_1 = W_1 + \mathrm{I}(\mathrm{I}(Y_2) + \mathrm{I}(Y_3)), \tag{11}$$
$$Y_2 = W_2 + \mathrm{I}(\mathrm{I}(Y_1) + \mathrm{I}(Y_3)), \tag{12}$$
$$Y_3 = W_3 + \mathrm{I}(\mathrm{I}(Y_1) + \mathrm{I}(Y_2)), \tag{13}$$

where $Y_i = \frac{y_{it} - c_i}{\beta_i}$, $W_i = \frac{w_{it} - c_i}{\beta_i}$, and $w_{it} = \boldsymbol{\alpha}_i'\mathbf{x}_{it} + u_{it}$, $i = 1, 2, 3$, and the time subscript has been suppressed for simplicity of notation.

Given that the system of equations (11) through (13) is symmetric, the reduced form will take the same form for each country and attention can be restricted to Y_1. The reduced form consists of the following subcases.

[A] if $W_2 > 0$ and/or $W_3 > 0$, then $Y_1 = W_1 + 1$;

[B] if $W_2 \leq -1$ and $W_3 \leq -1$, then $Y_1 = W_1$;

[C,D,E] if $W_2 \leq -1$ and $-1 < W_3 \leq 0$, then the system becomes

$$\begin{aligned}
Y_1 &= W_1 + \mathrm{I}(\mathrm{I}(Y_3)) \\
Y_2 &= W_2 + \mathrm{I}(\mathrm{I}(Y_1) + \mathrm{I}(Y_3)) \ \leq \ 0 \\
Y_3 &= W_3 + \mathrm{I}(\mathrm{I}(Y_1));
\end{aligned}$$

and Y_2 is not important for the solution of Y_1. The system reduces to a two equation system for Y_1 and Y_3. For the solution here regions B, D, and E in Equation (4) are relevant;

[F,G,H] if $-1 < W_2 \leq 0$ and $W_3 \leq -1$, then the solution is symmetric to the case above (C,D,E) as Y_3 is unimportant for the solution of Y_1, and the system reduces to a two equations system for Y_1 and Y_2;

[I,J,K] if $-1 < W_2 \leq 0$ and $-1 < W_3 \leq 0$ has three sub-cases

- **[I]** if $W_1 > 0$, then $Y_1 = W_1 + 1$;
- **[J]** if $-1 < W_1 \leq 0$ then this is the three country equivalent of the two country case with no unique solution, and $Y_1 = d_J W_1 + (1 - d_J)(W_1 + 1)$, where d_J is a selection parameter as in the case of two countries discussed in Section 2;

Table 9: Sources of the data

Variable	Country	Data series
Stock market index	UK	FT Ordinary ind. share price (MEI)
	USA	NYSE Composite (MEI)
	Japan	TSE Topix all shares (MEI)
Short interest rate	UK	T-bill rate (IFS: 60C)
	USA	T-bill rate (IFS 60C)
	Japan	Lending rate (IFS: 60P)
Industrial production	UK	not seasonally adj.(IFS: 66)
	USA	not seasonally adj.(IFS: 66)
	Japan	not seasonally adj.(IFS: 66)
Oil price		Average crude price (IFS: 176)

MEI indicates series taken from the OECD's Main Economic Indicator data base and IFS series taken from the IMF's International Financial Statistics data base, which are given with their respective codes.

[K] if $W_1 \leq -1$ then the system is still not unique as equations (12) and (13) reduce to the two country system with no unique solution, i. e. region E in Equation (4). Hence, $Y_1 = d_K W_1 + (1 - d_K)(W_1 + 1)$.

A system with $N = 4$ can be solved similarly for country i by considering the cases where at least one $W_j > 0$, $j \neq i$, the case where all $W_j \leq -1$, and the other cases can then be reduced to the system with $N = 3$ or solutions that are not unique. Systems with any N can be solved recursively this way.

B Appendix: The data

The stock market indices are taken from the OECD Main Economic Indicators data base and the explanatory variables from the IMF's International Financial Statistics data bases with details given in Table 9. For the share price I take the time series available from the OECD Main Economic Indicator data base, and in the case of the UK, the longer of the two time series. The short rate is the 3-month T-bill rate. In the case of Japan the T-bill rate is not available and I take the lending rate, which unlike other Japanese interest rates is not zero in the deflation period in the 1990's—in that case the growth rate would not be defined. Industrial production is taken non-seasonally adjusted to avoid problems of endogeneity. The oil price is the average crude oil price.

Table 10: ADF test statistics

Variable	UK	USA	Japan
Stock market index	6.313	5.816	5.314
Industrial prod.	6.115	4.257	4.427
Short rate	6.452	3.832	6.391
Oil price	6.049		

Each variable is in growth rates, and the absolute test statistic of Augmented Dickey-Fuller (ADF) test are reported, where the optimal lag length was determined using AIC.

The data are transformed into growth rates

$$\Delta x_t = \frac{x_t - x_{t-l}}{x_{t-l}},$$

where x_t is the variable in levels and l is 3 for the stock market index, the short rate and oil prices and 12 for industrial production. The annual growth rate for industrial production is necessary in order to eliminate seasonal variation in industrial production, which is not present in the other time series.

Table 10 contains the results testing for a unit root in the data using the ADF test (Dickey and Fuller 1979), where the optimal lag length was determined using AIC. The results show that a unit root can clearly be rejected for all the time series.

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