Econometric Issues in the Analysis of Contagion*

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This version: March 7, 2006

Abstract

This paper presents a canonical, econometric model of contagion and investigates the conditions under which contagion can be distinguished from interdependence. In a two-market set up it is shown that for a range of fundamentals the solution is not unique, and for sufficiently large values of the contagion coefficients it has interesting bifurcation properties with bimodal density functions. The identification of contagion requires that the equations for the individual markets contain market specific regressors. This sheds doubt on the general validity of the correlation based tests of contagion recently proposed in the literature which do not involve any market specific variables. Furthermore, we show that ignoring endogeneity and interdependence can introduce an upward bias in the estimate of the contagion coefficient, and using Monte Carlo experiments we further show that this bias could be substantial. Finally, we analyse data on European interest rates spreads during the ERM and find a clear asymmetry in the contagion effects of sharp rises and falls; with only the former having some statistically significant effects.

JEL Classifications: C10, C123, G10, G15.
Keywords: Contagion, Interdependence, Identification, Financial Crises.

*We would like to thank Mardi Dungey, Teresa Leitão, Roger Moon, Geert Rider, Til Schuermann and participants at the CERF Seminar, "International Financial Contagion", University of Cambridge, and seminars at the University of Southern California and the Tinbergen Institute, Amsterdam, for helpful comments. Constructive comments and suggestions by two anonymous referees and the editor (Peter Ireland) have also been most helpful to us. The main part of the research for this paper was done while the second author was a graduate student at the University of Cambridge. The views expressed in this paper are those of the authors and do not necessarily reflect those of the De Nederlandsche Bank.

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1 Introduction

It has been frequently observed that financial crises appear in clusters. There exists now a large body of literature that attempts to distinguish between contagion and interdependence. This literature has been reviewed by Dornbusch, Park, and Claessen (2000), Pericoli and Sbracia (2002), and Dungey, Fry, Gonzalez-Hermosillo and Martin (2005). The theoretical literature on financial crises considers a number of reasons for crises to appear in clusters. Masson (1999) identifies three categories under which the different theories can be subsumed. First, the theory of "monsoonal effects" suggests that financial crises appear to be contagious because underlying macroeconomic variables are correlated. Second, financial crises may be transmitted between countries via "spill-overs": a crisis affects another country through external links such as trade. Finally, the theory of "pure contagion" holds that the market jumps from a "good" to a "bad" equilibrium.

The first two cases, monsoonal effects and spill-overs, are examples of interdependence. Crises resulting from interdependence could, in principle, be predictable using macroeconomic fundamentals. If the interdependence during non-crises periods is known, the effect of a financial crisis in one country on the likelihood of a crisis in another country can be evaluated. The third case, jumps between equilibria, is what we refer to as contagion in this paper: a largely unpredictable, higher correlation during crises times compared to normal times. This definition of contagion means that a crisis in one country increases the likelihood of a crisis in another country over and above what would be implied by the interdependence that prevails between these countries in non-crises times. This definition corresponds to that given, for example, by Forbes and Rigobon (2001, 2002).

The distinction between contagion and interdependence has important implications. Investors need to adjust their portfolios accordingly if markets have a higher correlation during crises as diversification of portfolios across markets might be less useful than anticipated if based on correlations in tranquil times. Equally, the policy responses to a crisis will depend on the perceived nature of transmission of shocks across the financial markets. If the cause of a crisis is a random jump between equilibria, i. e. contagion, policy intervention could be effective. In contrast, if a crisis spreads to other markets because the fundamentals are correlated, then policy-makers are less likely to be able to prevent a crisis from spreading.

In this paper we propose a canonical model of contagion that allows for all the three different causes of crises: first, country or market specific shocks, second, common observed or unobserved factors, i. e. interdependence, and, third, higher correlation during crises times, i. e. contagion. Furthermore, we characterise the solution of the model and we find that the solution is not unique for a range of fundamentals. For sufficiently large values of the contagion coefficients the solution has interesting bifurcation properties with bimodal density functions.

In section 3 we discuss the problems of identification and estimation of the contagion coefficients in the canonical model. The estimation is shown to be an example of the general problem of inference in the non-linear simultaneous equation models. To identify contagion effects in the presence of inter-dependencies the equations for the individual markets or countries must contain country (market) specific variables. The extension to a multi-country or -market model is considered in section 4.

In view of our discussion of the canonical model and its properties we reconsider the extant empirical literature on contagion in section 5. A set of papers examines contagion of financial markets by testing for higher correlation between markets during crises times, inter alia, King and Wadhwani (1990), Boyer, Gibson, and Loretan (1999), Loretan and English (2000), Forbes and Rigobon (2002), Bae, Karolyi and Stulz (2003), and Corsetti, Pericoli and Sbracia (2005). However, pure correlation-based tests for contagion cannot be valid. Country specific regressors are needed to distinguish contagion from interdependence. The correlation based tests of contagion recently proposed in the literature attempt to overcome the identification problem by assuming that, first, the crises periods can be identified *a priori*, and that, second, such episodes are sufficiently prolonged and contiguous so that cross-country (market) correlations during crisis and non-crisis periods can be consistently estimated and compared. These are strong assumptions that are unlikely to hold in practice and their implementation tends to be subject to a sample selection bias. Such correlation analyses are *ex post* in nature and are therefore not helpful if the focus of the analysis is to develop an early warning system for policy use.

Favero and Giavazzi (2002) develop a test of contagion using a simultaneous equation framework to distinguish interdependence from contagion, but continue to rely on ex post identification of crisis from non-crisis periods. They also require that the identified set of crisis periods (dummies) can be classified into those that are common to all markets under consideration and those that are market specific. The test of contagion is then carried out by checking the significance of country specific crisis dummies (treated as predetermined) in equations for other countries. Favero and Giavazzi's framework is closer to our modelling approach, but is still subject to the sample selection bias, and cannot be used for forecasting or for the development of early warning systems.

A second set of papers has been based on the literature on the macroeconomic causes of currency crises, for example Eichengreen, Rose, and Wyplosz (ERW) (1996), Esquivel and Larraín (1998), Kruger, Osakwe, and Page (1998), Stone and Weeks (2001), and Kumar, Moorthy, and Perraudin (2002). We show that ignoring the endogeneity of the contagion indicator and/or interdependence of the error terms can introduce an upward bias in the estimate of the contagion coefficient, and using Monte Carlo experiments we further show that this bias could be substantial. Our simulations also suggest that the contagion coefficient of 0.54 obtained from pooled probit estimation of ERW's model could be due to neglected interdependencies rather than contagion.

In section 6 we estimate a two-sided version of the contagion model advanced in this paper using weekly observations on three month interest rates spreads (relative to German rates) for seven European economies, analysed previously by Favero and Giavazzi (2002). In our set up identification of crises is endogenized and their effects are estimated simultaneously with the coefficients of interdependence of the spreads in normal periods. We find a clear asymmetry in the contagion effects of sharp rises and sharp falls in interest rates spreads; with only the former having some statistically significant effects.

2 A Canonical Model of Contagion: A Two-Country Framework

Consider the following relations

$$y_{1t} = \boldsymbol{\delta}_1'\mathbf{z}_t + \boldsymbol{\alpha}_1'\mathbf{x}_{1t} + \beta_1 \mathrm{I}(y_{2t} - c_2\sigma_{2,t-1}) + u_{1t} \tag{1}$$

$$y_{2t} = \boldsymbol{\delta}_2'\mathbf{z}_t + \boldsymbol{\alpha}_2'\mathbf{x}_{2t} + \beta_2 \mathrm{I}(y_{1t} - c_1\sigma_{1,t-1}) + u_{2t}, \tag{2}$$

where y_{it} is a performance indicator for country $i = 1, 2$, $t = 1, \ldots, T$, u_{1t} and u_{2t} are serially uncorrelated errors with zero means, conditional variances $\sigma^2_{u1,t-1}$ and $\sigma^2_{u2,t-1}$ and a non-zero correlation coefficient ρ. While it is in principle possible to allow for time variations in ρ, such a generalisation could obscure the properties of the correlation between y_{1t} and y_{2t}. We show below that $\mathrm{Corr}(y_{1t}, y_{2t})$ could be time varying even if ρ is not. The regressors, $\mathbf{x}_{it}$, are $k_i \times 1$ country-specific observed factors assumed to be pre-determined and distributed independently of u_{jt} for all i and j. Country-specific dynamics can be allowed for by including $y_{i,t-1}, y_{i,t-2}, \ldots$ in $\mathbf{x}_{it}$. The $s \times 1$ vector $\mathbf{z}_t$ contains pre-determined observed common factors, such as international oil prices. $\mathrm{I}(A)$ is an indicator function that takes the value of unity if $A > 0$ and zero otherwise,

$$\sigma^2_{i,t-1} = \mathrm{Var}\left(y_{it} \mid \Omega_{t-1}\right),$$

where Ω_{t-1} is the information available at time $t-1$.

Examples of performance indicators include stock market returns used by Forbes and Rigobon (2002) and Corsetti, Pericoli and Sbracia (2005), and the index of "exchange market pressure" employed by Eichengreen, Rose and Wylosz (1996), which is a weighted average of exchange rate depreciation, interest rates differential and international reserves ratios. We are assuming that y_{it} is defined in such a way that a crisis is associated with extreme positive values of y_{it}, and $c_i > 0$.

In this set up interdependence is captured through non-zero values of ρ, and is distinguished from contagion effects characterised by non-zero values of β_i.

- It is assumed that contagion takes place only at times of crises, whilst interdependence is the result of normal market interactions.
- Country i is said to be in crisis if the performance index, y_{it}, rises above a threshold value c_{it}.
- Contagion is said to occur if a crisis in country 2 increases the probability of a crisis in country 1 over and above the usual market interactions, and *vice versa.*
- To test for contagion we first need to establish conditions under which the contagion coefficients, β_i, can be identified. Once such conditions are met, a test of contagion in country i can be carried out by testing $\beta_i = 0$ against the one-sided alternatives, $\beta_i > 0$ allowing for the possibility of non-zero ρ.

The above framework can be readily generalised to deal with both extremes simultaneously,

$$y_{it} = \boldsymbol{\delta}_i'\mathbf{z}_t + \boldsymbol{\alpha}_i'\mathbf{x}_{it} + \beta_{iU}\mathrm{I}(y_{jt} - c_{jU}\sigma_{j,t-1}) + \beta_{iL}\mathrm{I}(-y_{jt} - c_{jL}\sigma_{j,t-1}) + u_{it},$$

for $i = 1, 2$, where β_{iU} and β_{iL} now refer to contagion effects on the upper and the lower tails and $c_{jU}\sigma_{j,t-1}$ and $c_{jL}\sigma_{j,t-1}$ are the associated thresholds with $c_{jU} \geq 0$ and $c_{jL} \geq 0$. It is clear that only one of the indicators can be triggered at a time.

Another possible generalisation would be to consider endogenous switches in the slope coefficients of the fundamentals (δ_i, α_i) as well as in the intercepts. More specifically, we could have, for example,

$$y_{it} = \boldsymbol{\delta}_i'\mathbf{z}_t + \boldsymbol{\alpha}_i'\mathbf{x}_{it} + (\beta_i + \gamma_i x_{it})\mathrm{I}(y_{jt} - c_j\sigma_{j,t-1}) + u_{it}.$$

Here we shall focus on the relatively simple model defined in (1) and (2), but we conjecture that our approach and arguments can be readily extended to the more general case.

2.1 Solution and Possibility of Multiple Equilibria

Setting

$$w_{it} = \boldsymbol{\delta}_i'\mathbf{z}_t + \boldsymbol{\alpha}_i'\mathbf{x}_{it} + u_{it},$$

we re-write (1) and (2) as

$$y_{1t} = w_{1t} + \beta_1 \mathrm{I}(y_{2t} - c_2), \tag{3}$$
$$y_{2t} = w_{2t} + \beta_2 \mathrm{I}(y_{1t} - c_1), \tag{4}$$

where to simplify the notations and without loss of generality we abstract from the (possibly) time varying nature of the thresholds.

This is a system of non-linear and non-differentiable simultaneous equations and has a simple unique solution when either β_1 or β_2 is zero. For example, suppose that $\beta_2 = 0$. Then the solution is given by

$$y_{1t} = w_{1t} + \beta_1 \mathrm{I}(y_{2t} - c_2), \tag{5}$$
$$y_{2t} = w_{2t}. \tag{6}$$

When both contagion coefficients are positive the equation system (3) and (4) can be equivalently written as

$$Y_{1t} = W_{1t} + \mathrm{I}(Y_{2t}), \tag{7}$$
$$Y_{2t} = W_{2t} + \mathrm{I}(Y_{1t}), \tag{8}$$

where

$$Y_{it} = \frac{y_{it} - c_i}{\beta_i},\ W_{it} = \frac{w_{it} - c_i}{\beta_i}. \tag{9}$$

To solve this simplified system we shall consider the following five mutually exclusive regions in the (W_{1t}, W_{2t}) plane—see also Figure 1:

Region A: $W_{2t} > 0$,

Region B: $-1 < W_{2t} \leq 0$ and $W_{1t} > 0$,

Region C: $W_{2t} \leq -1$,

Region D: $-1 < W_{2t} \leq 0$ and $W_{1t} < -1$,

Region E: $-1 < W_{2t} \leq 0$ and $-1 < W_{1t} \leq 0$.

It is now easily verified that in regions A and B, the solution for Y_{1t} is unique and is given by

$$Y_{1t}^* = 1 + W_{1t}, \tag{10}$$

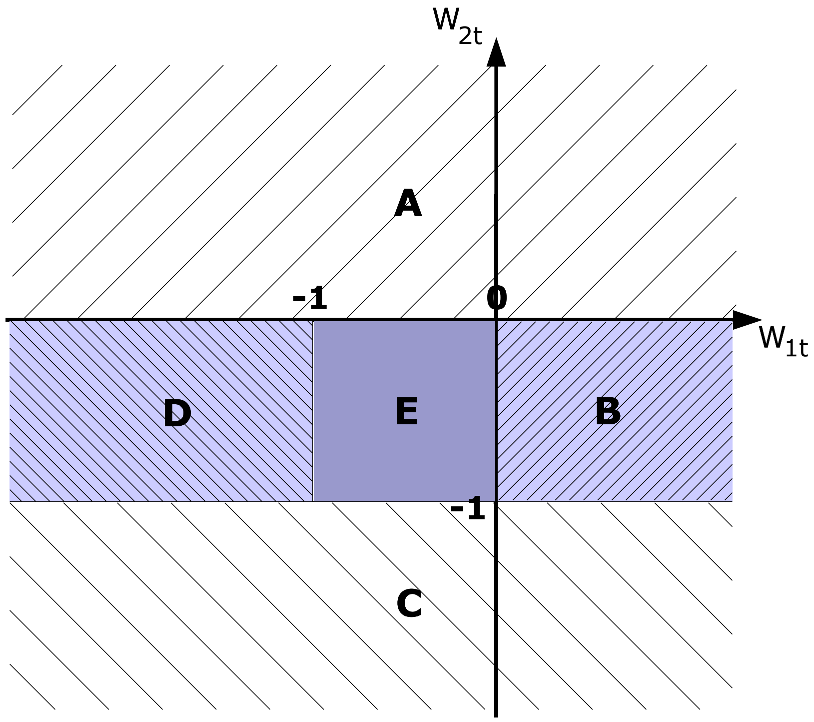
and, similarly, in regions C and D the solution is unique and is given by

$$Y_{1t}^* = W_{1t}. \tag{11}$$

However, in region E the solution is not unique. For example, for $W_{1t} = -1/2$, and $W_{2t} = -1/3$, there are two possible solutions for $\mathbf{Y}_t = (Y_{1t}, Y_{2t})'$ given by

$$\mathbf{Y}_t^a = \begin{pmatrix} -1/2 \\ -1/3 \end{pmatrix} \text{ and } \mathbf{Y}_t^b = \begin{pmatrix} 1/2 \\ 2/3 \end{pmatrix}.$$

Figure 1: Regions of W_{1t} and W_{2t}



This problem of coherency has been discussed, for example, by Gourieroux, Laffont, and Monfort (1980). In the case of systems of binary choice equations Lewbel (2006) shows that coherency requires the system to be triangular in each period, although the direction of causality can vary across the periods. This solution, however, requires information beyond that contained in the model.

Here we extend the model by using the index d_t to designate the choice of the solution when $-1 < W_{it} \leq 0$ we have

$$Y_{it}^*(d_t) = d_t W_{it} + (1 - d_t)(1 + W_{it}), \text{ for } i = 1, 2, \tag{12}$$

where the "favourable" solution occurs if $d_t = 1$, and the "unfavourable" solution occurs if $d_t = 0$. Notice that in the present set up the crisis (unfavourable outcome) is associated with the upper tail (large positive values). It is clear from Equation (12) that the distribution of $Y_{it}^*(d_t)$ is a mean mixture of distributions with d_t as the selection parameter, and $d_t \sim \text{Bernoulli}(\pi)$, where π is the probability of W_{it} being chosen in the mixture.

This is an interesting example where non-uniqueness arises only if the fundamentals (as measured by W_{it}) for both countries (markets) are favourable but weak (in relation to the threshold values). This appears similar to the notion of weak fundamentals used by Sachs, Tornell and Velasco (1996). It is also reasonable to expect that the correlation of Y_{1t} and Y_{2t} would be higher if the unfavourable solution is chosen as compared to the favourable one. Simulation results reported below bear this out. However, we leave a more detailed modelling of d_t for future research.

Collecting the various components of the solution given by (10) to (12) we have

$$
\begin{aligned}
Y_{1t} \;=\; & (1+W_{1t})\,\mathrm{I}\,(W_{2t}) && \text{(Region A)} \\
& +(1+W_{1t})\;\mathrm{I}\,(-W_{2t})\;\mathrm{I}(1+W_{2t})\,\mathrm{I}(W_{1t}) && \text{(Region B)} \\
& +W_{1t}\,\mathrm{I}(-1-W_{2t}) && \text{(Region C)} \\
& +W_{1t}\,\mathrm{I}\,(-W_{2t})\;\mathrm{I}(1+W_{2t})\,\mathrm{I}(-1-W_{1t}) && \text{(Region D)} \\
& +Y_{1t}^{*}(d_t)\,\mathrm{I}\,(-W_{2t})\;\mathrm{I}(1+W_{2t}) && \text{(Region E)} \\
& \times\mathrm{I}(-W_{1t})\,\mathrm{I}(1+W_{1t})
\end{aligned}
\tag{13}
$$

and by symmetry

$$
\begin{aligned}
Y_{2t} = {} & (1+W_{2t})\,\mathrm{I}\,(W_{1t}) \\
& +(1+W_{2t})\,\mathrm{I}\,(-W_{1t})\;\mathrm{I}(1+W_{1t})\mathrm{I}(W_{2t}) \\
& +W_{2t}\,\mathrm{I}(-1-W_{1t}) \\
& +W_{2t}\,\mathrm{I}\,(-W_{1t})\;\mathrm{I}(1+W_{1t})\,\mathrm{I}(-1-W_{2t}) \\
& +Y_{2t}^{*}(d_t)\,\mathrm{I}\,(-W_{1t})\;\mathrm{I}(1+W_{1t})\,\mathrm{I}(-W_{2t})\,\mathrm{I}(1+W_{2t}).
\end{aligned}
\tag{14}
$$

In terms of the original variables we obtain

$$
y_{it}^{*}=\beta_i Y_{it}^{*}+c_{it},\;\text{ for } i=1,2. \tag{15}
$$

It is important that the above solution is valid even if $y_{i,t-1}$, $y_{i,t-2}$, are included amongst of the individual-specific regressors, $\mathbf{x}_{it}$. This feature considerably enhances the relevance of the model to the analysis of financial markets that show a mild degree of short term over-shooting.

It is clear that y_{1t} and y_{2t} will be correlated even if w_{1t} and w_{2t} are independently distributed, i.e. for values of $\beta_i > 0$, $\mathrm{Corr}(y_{1t}, y_{2t}) > 0$ even when $\mathrm{Corr}(w_{1t}, w_{2t}) = 0$. For example, consider the simple case of Equations (5) and (6) where $\beta_2 = 0$, $\beta_1 > 0$, and w_{1t} and w_{2t} are independently distributed. In this case

$$
\mathrm{Cov}(y_{1t},y_{2t})=\beta_1\left[1-\mathrm{F}_2(c_2)\right]\left\{\mathrm{E}\left(w_{2t}-c_2\mid w_{2t}>c_2\right)-\mathrm{E}\left(w_{2t}-c_2\right)\right\},
$$

and

$$
\mathrm{Corr}(y_{1t},y_{2t})=\frac{\beta_1\left[1-\mathrm{F}_2(c_{2t})\right]\left\{\mathrm{E}\left(w_{2t}-c_2\mid w_{2t}>c_2\right)-\mathrm{E}\left(w_{2t}-c_2\right)\right\}}{\sqrt{\mathrm{Var}(w_{2t})\left\{\mathrm{Var}(w_{1t})+\beta_1^2\mathrm{F}_2(c_2)\left[1-\mathrm{F}_2(c_2)\right]\right\}}},
$$

Table 1: Moments of the distribution of $\mathbf{y}_t$

	$\pi = 1$ $(d_t = 1)$				$\pi = 0$ $(d_t = 0)$			
β	$\bar{y}_1$	$\sigma(y_1)$	Kurt	Corr	$\bar{y}_1$	$\sigma(y_1)$	Kurt	Corr
				$\rho = 0$				
0.5	0.028	1.00	0.08	0.120	0.030	1.01	0.07	0.127
1.0	0.063	1.05	0.43	0.238	0.107	1.11	0.15	0.319
2.0	0.161	1.24	1.96	0.457	0.863	1.69	−1.13	0.706
				$\rho = 0.5$				
0.5	0.033	1.04	0.19	0.576	0.037	1.05	0.15	0.582
1.0	0.073	1.12	0.69	0.641	0.146	1.21	0.12	0.693
2.0	0.172	1.34	1.88	0.734	0.977	1.82	−1.31	0.854

"Kurt" denotes Kurtosis-3 of the distribution of y_{1t} and "Corr" the correlation between y_{1t} and y_{2t}.

where $\mathrm{F}_2(x)$ is the cumulative distribution function of w_{2t}. In the extreme value literature, $\mathrm{E}(w_{2t} - c_2 \mid w_{2t} > c_2)$ is known as the mean excess function of w_{2t}, see for example Embrechts, Klüppelberg and Mikosch (1997). This result provides support for the hypothesis that the degree of the dependence of y_{1t} and y_{2t} is an increasing function of the degree of the fat-tailedness of the w_{2t} process. For $w_{it} \sim \mathrm{N}(0,1)$,

$$\mathrm{Corr}(y_{1t}, y_{2t}) = \frac{\beta_1 \left[1 - \Phi(c_2)\right] \left\{\mathrm{E}\left(w_{2t} \mid w_{2t} > c_2\right)\right\}}{\sqrt{1 + \beta_1^2 \Phi(c_2) \left[1 - \Phi(c_2)\right]}} > 0, \text{ for } \beta_1 > 0,\ c_2 > 0.$$

2.2 Some Numerical Results

Suppose that $c_{it} = 1.64$ (that corresponds to the upper 95% tail of the standard normal), let $\beta_1 = \beta_2 = \beta$, and

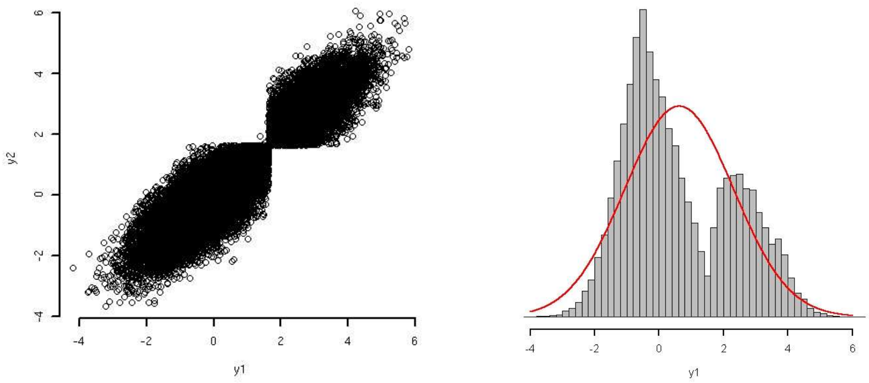
$$\begin{pmatrix} w_{1t} \\ w_{2t} \end{pmatrix} \sim \mathrm{N}\left(\mathbf{0}, \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}\right).$$

Using these parameters we can sample the dependent variables and investigate their properties for different values of the contagion coefficient β. The results reported below are based on 30,000 sampled values of y_{1t} and y_{2t}.

Table 1 reports the moments of y_{1t} and the correlation of y_{1t} and y_{2t} under the assumption that only one of the mixture distributions is visited. Note, however, that due to the symmetry of the model the reported moments also apply to y_{2t}. On the left side of the table the results for $\pi = 1$ are reported and on the right side the results for $\pi = 0$.

Rather than choosing only one part of the mixture in (15) one can also consider intermediate cases where both parts of the mixture are visited. Below we set $\pi = 0.5$ by sampling $d_t = \mathrm{I}(s_t)$ where $s_t \sim \mathrm{N}(0,1)$. In this case

Figure 2: Scatter plot of y_1 on y_2, and histogram of y_1 with normal curve ($\beta = 2$, $\rho = 0.8$, $\pi = 0.5$)



one obtains very pronounced bimodal distributions for y_{it}^*. A clear polar separation of solutions emerges when β is large, as can be seen in Figures 2 for $\beta = 2$ and $\rho = 0.8$. More dramatic pictures can be obtained for larger values of β as in Figure 3. These parameter values are chosen for illustrative purposes and we do not expect to observe such extreme phenomena in practice. For small values of β the polarisation is very slight and cannot be revealed by visual inspection. This can be seen in Figures 4, which display the results for $\beta = 0.5$ and $\rho = 0.5$.

3 Identification and Estimation of the Contagion Coefficients

The system of equations (1) and (2) represent a two-equation non-linear simultaneous equation model, which has been studied extensively in the econometric literature as reviewed, for example, by Amemiya (1985). The above equation system whilst non-linear in the endogenous variables, $\mathbf{y}_t = (y_{1t}, y_{2t})'$, is linear in the parameters for known threshold values, c_1 and c_2. This somewhat simplifies the identification and estimation problems. In what follows we focus on this relatively simple case by assuming that c_1 and c_2 are known and that the variances $\sigma_{i,t-1}$ are time invariant and can be absorbed in c_i. The non-uniqueness of the solution itself is no impediment to identification and/or consistent estimation of the unknown parameters. As in the case of simultaneous equation models, it is possible to consistently estimate the parameters of a single equation in a system without necessarily having to fully specify the system of equations. An additional equation for d_t, is not necessary for the consistent estimation of the contagion coefficients

Figure 3: Scatter plot of y_1 on y_2, and histogram of y_1 with normal curve ($\beta = 3.5$, $\rho = 0.8$, $\pi = 0.5$)

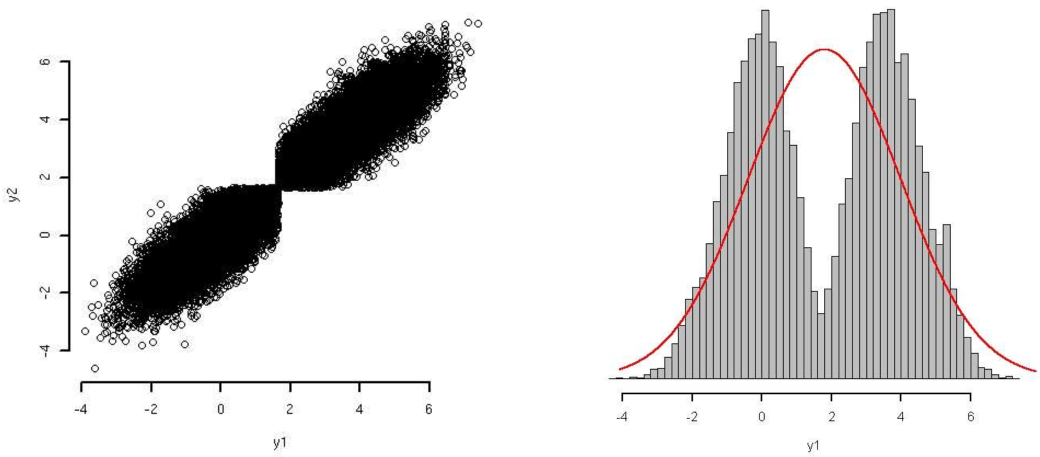
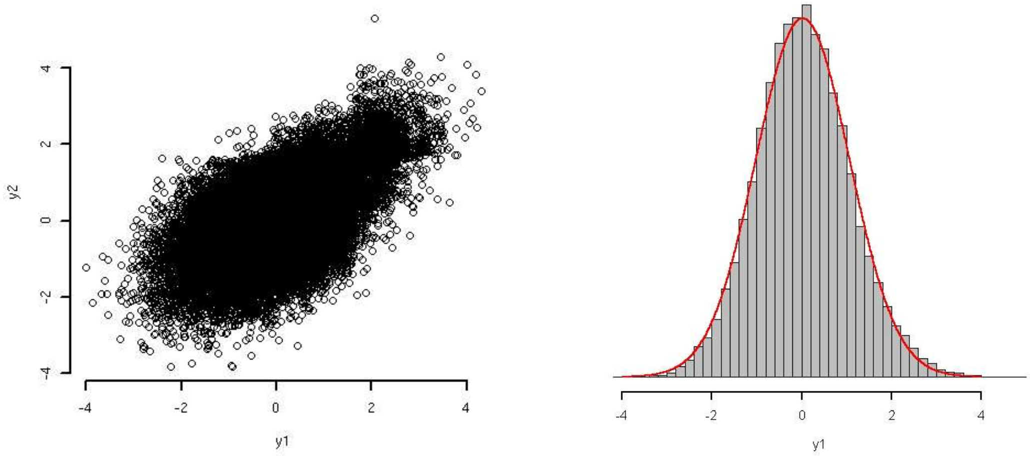


Figure 4: Scatter plot of y_1 on y_2, and histogram of y_1 with normal curve ($\beta = 0.5$, $\rho = 0.5$, $\pi = 0.5$)



β_i, for example. However, the identification problem becomes much more complicated and poses new challenges if the focus of the analysis is also on the identification of the d_t process itself. This is beyond the scope of the present paper and will not be addressed. Instead, our focus will be on identification and consistent estimation of the contagion coefficients.

3.1 Inconsistency of the OLS Estimators

Consider the Ordinary Least Squares (OLS) regressions of y_{it} on $\mathbf{z}_t$, $\mathbf{x}_{i,t}$, $\mathrm{I}(y_{jt} - c_j)$, for $i, j = 1, 2$ and for simplicity suppose that the two equations only contain one country-specific regressor each and assume that these regressors (x_{1t}, x_2) are strictly exogenous, stationary, and distributed independently of the errors, u_{1t} and u_{2t},

$$y_{1t} = \alpha_1 x_{1t} + \beta_1 \mathrm{I}(y_{2t} - c_2) + u_{1t}, \tag{16}$$

$$y_{2t} = \alpha_2 x_{2t} + \beta_2 \mathrm{I}(y_{1t} - c_1) + u_{2t}, \tag{17}$$

where

$$\begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} | x_{1t}, x_{2t} \sim N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{u1}^2 & \rho\sigma_{u1}\sigma_{u2} \\ \rho\sigma_{u1}\sigma_{u2} & \sigma_{u2}^2 \end{pmatrix} \right].$$

Suppose also that probability of crisis occurring in either of the two countries are neither zero nor unity, namely

$$T^{-1} \sum_{t=1}^{T} \mathrm{I}(y_{jt} - c_j) \to \pi_j, \text{ where } 1 > \pi_j > 0, \tag{18}$$

which is shown to be true for errors with unbounded support in Appendix A. We also have

$$T^{-1} \sum_{t=1}^{T} x_{jt}^2 \to \sigma_{xj}^2 > 0, \tag{19}$$

$$T^{-1} \sum_{t=1}^{T} x_{jt} u_{it} \to 0, \text{ for } i, j = 1, 2. \tag{20}$$

The OLS estimator of β_1 is given by

$$\hat{\beta}_1 = \left(\mathbf{d}_2' \mathbf{M}_1 \mathbf{d}_2\right)^{-1} \mathbf{d}_2' \mathbf{M}_1 \mathbf{y}_1,$$

where $\mathbf{d}_2 = (\mathrm{I}(y_{21} - c_2), \mathrm{I}(y_{22} - c_2), \ldots, \mathrm{I}(y_{2T} - c_2))'$, $\mathbf{M}_1 = \mathbf{I}_T - \mathbf{x}_1(\mathbf{x}_1'\mathbf{x}_1)^{-1}\mathbf{x}_1'$, $\mathbf{x}_1 = (x_{11}, x_{12}, \ldots, x_{1T})'$, and $\mathbf{y}_1 = (y_{11}, y_{12}, \ldots, y_{1T})'$. Furthermore,

$$T^{-1}\left(\mathbf{d}_2' \mathbf{M}_1 \mathbf{d}_2\right) = T^{-1} \sum_{t=1}^{T} \mathrm{I}(y_{2t} - c_2) - \frac{\left[T^{-1} \sum_{t=1}^{T} \mathrm{I}(y_{2t} - c_2) x_{1t}\right]^2}{T^{-1} \sum_{t=1}^{T} x_{1t}^2},$$

and $T^{-1}(\mathbf{d}_2'\mathbf{M}_1\mathbf{d}_2)$ tends to a non-zero constant, $\omega_{22} > 0$. This is easily seen in the simple case where $x_{1t} = 1$ for all t. In this case $T^{-1}(\mathbf{d}_2'\mathbf{M}_1\mathbf{d}_2)$ converges to $\pi_2(1-\pi_2) > 0$. Hence

$$\operatorname*{plim}_{T\to\infty}\left(\hat{\beta}_1\right) = \beta_1 + \frac{\operatorname*{plim}_{T\to\infty}\left(\frac{\mathbf{d}_2'\mathbf{M}_1\mathbf{u}_1}{T}\right)}{\omega_{22}}.$$

where $\mathbf{u}_1 = (u_{11}, u_{12}, \ldots, u_{1T})'$. Also under our assumptions (see in particular (19) and (20))

$$\begin{aligned}\operatorname*{plim}_{T\to\infty}\left(\frac{\mathbf{d}_2'\mathbf{M}_1\mathbf{u}_1}{T}\right) &= \operatorname*{plim}_{T\to\infty}\left(\frac{\mathbf{d}_2'\mathbf{u}_1}{T}\right) - \frac{\operatorname*{plim}_{T\to\infty}\left(\frac{\mathbf{d}_2'\mathbf{x}_1}{T}\right)\operatorname*{plim}_{T\to\infty}\left(\frac{\mathbf{x}_1'\mathbf{u}_1}{T}\right)}{\sigma_{x1}^2} \\ &= \mathrm{E}\left[u_{1t}\mathrm{I}(y_{2t}-c_2)\right],\end{aligned}$$

and

$$\operatorname*{plim}_{T\to\infty}\left(\hat{\beta}_1\right) = \beta_1 + \frac{\mathrm{E}\left[u_{1t}\mathrm{I}(y_{2t}-c_2)\right]}{\omega_{22}}.$$

In general, $\mathrm{E}\left[u_{1t}\mathrm{I}(y_{2t}-c_2)\right] \neq 0$, and the OLS estimator of β_1 is inconsistent. The sign and the magnitude of the inconsistency of $\hat{\beta}_1$ depends on β_2 and ρ. The OLS estimator of β_1 is consistent only if $\beta_2 = \rho = 0$, namely if the contagion model is recursive (triangular) and there are no interdependencies through the errors. To see this consider the relatively simple case where $\beta_2 = 0$, and note that under normally distributed errors we have

$$u_{1t} = \rho\left(\frac{\sigma_{u1}}{\sigma_{u2}}\right)u_{2t} + v_t, \tag{21}$$

where u_{2t} and v_t are independently distributed. Note also that v_t is distributed independently of x_{1t} and x_{2t} and has a zero mean. In this case

$$\begin{aligned}\mathrm{E}\left[u_{1t}\mathrm{I}(y_{2t}-c_2)\right] &= \mathrm{E}\left[u_{1t}\mathrm{I}(\alpha_2 x_{2t}+u_{2t}-c_2)\right] \\ &= \rho\left(\frac{\sigma_{u1}}{\sigma_{u2}}\right)\mathrm{E}\left[u_{2t}\mathrm{I}(\alpha_2 x_{2t}+u_{2t}-c_2)\right] + \mathrm{E}\left[v_t\mathrm{I}(\alpha_2 x_{2t}+u_{2t}-c_2)\right].\end{aligned}$$

Since v_t is distributed independently of x_{2t} and u_{2t}, then conditional on x_{2t} and u_{2t}

$$\mathrm{E}\left[v_t\mathrm{I}(\alpha_2 x_{2t}+u_{2t}-c_2)\left|u_{2t},x_{2t}\right.\right] = \mathrm{I}(\alpha_2 x_{2t}+u_{2t}-c_2)\mathrm{E}\left(v_t\left|u_{2t},x_{2t}\right.\right) = 0,$$

and

$$\mathrm{E}\left[u_{1t}\mathrm{I}(y_{2t}-c_2)\right] = \rho\left(\frac{\sigma_{u1}}{\sigma_{u2}}\right)\mathrm{E}\left[u_{2t}\mathrm{I}(\alpha_2 x_{2t}+u_{2t}-c_2)\right].$$

The following lemma shows that when $\rho > 0$, and $\beta_2 = 0$, then $\mathrm{E}[u_{2t}\mathrm{I}(y_{2t} - c_2)] > 0$, and $\hat{\beta}_1$ will be a consistent estimator of β_1 if and only if $\rho = 0$. The direction of the bias is upward when $\rho > 0$, and downward if $\rho < 0$.

Lemma 1 *Suppose $\beta_2 = 0$, and conditional on x_{2t}, u_{2t} is normally distributed, then* $\mathrm{E}\left[u_{2t}\mathrm{I}(y_{2t} - c_2)\right] > 0$ *if* $\rho > 0$.

Proof. Under $\beta_2 = 0$, $u_{2t}\mathrm{I}(y_{2t} - c_2) = u_{2t}\mathrm{I}(\alpha_2 x_{2t} + u_{2t} - c_2) = u_{2t}^*$, where

$$u_{2t}^* = \begin{cases} u_{2t} & \text{if } u_{2t} > c_2 - \alpha_2 x_{2t}, \\ 0 & \text{otherwise.} \end{cases}$$

Conditional on x_{2t}, noting that by assumption x_{2t}, and u_{2t} are independently distributed we have,

$$\mathrm{E}\left(u_{2t}^* \left| x_{2t}\right.\right) = \Pr(u_{2t} > c_2 - \alpha x_{2t} \left| x_{2t}\right.)\mathrm{E}\left(u_{2t} | u_{2t} > c_2 - \alpha_2 x_{2t},\ x_{2t}\right).$$

But

$$\mathrm{E}\left(u_{2t} | u_{2t} > c_2 - \alpha_2 x_{2t},\, x_{2t}\right) = \frac{\sigma_{u2}\phi\left(\frac{c_2 - \alpha_2 x_{2t}}{\sigma_{u2}}\right)}{\Pr(u_{2t} > c_2 - \alpha_2 x_{2t},\ x_{2t})}.$$

and, hence,

$$\mathrm{E}\left(u_{2t}^* \left| x_{2t}\right.\right) = \sigma_{u2}\phi\left(\frac{c_2 - \alpha_2 x_{2t}}{\sigma_{u2}}\right),$$

Since $\phi\left(\frac{c_2 - \alpha_2 x_{2t}}{\sigma_{u2}}\right) > 0$ for all values of x_{2t}, we also have that

$$\mathrm{E}\left(u_{2t}^*\right) = \mathrm{E}\left[u_{2t}\mathrm{I}(y_{2t} - c_2)\right] > 0.$$

■

Consider now the general case where $\rho > 0$ and $\beta_2 > 0$, and note that in this case (using (21)) we have

$$\mathrm{E}\left[u_{1t}\mathrm{I}(y_{2t} - c_2)\right] = \rho\left(\frac{\sigma_{u1}}{\sigma_{u2}}\right)\mathrm{E}\left[u_{2t}\mathrm{I}(Y_{2t})\right] + \mathrm{E}\left[\varepsilon_{1t}\mathrm{I}(Y_{2t})\right], \tag{22}$$

where Y_{2t} is given by the solution (14), which takes either the value of W_{2t} or $1 + W_{2t}$. The probability of whether the solution is W_{2t} or $1 + W_{2t}$ depends, in a complicated manner, on the probability of W_{1t} and W_{2t} falling in the regions A,B,C, D, and E, and the probability of a particular solution being selected if W_{1t} and W_{2t} fall in region E. In Appendix B we give results from Monte Carlo experiments, which show that the expectation is positive for a wide range of values of $\beta_1, \beta_2, \alpha_1, \alpha_2$, and ρ. Therefore, unless $\beta_2 = \rho = 0$, the OLS estimator of β_1 will be inconsistent. The large sample bias will be upward when $\rho > 0$ and $\beta_1 > 0$.

3.2 Consistent Estimation of the Contagion Coefficients

Consistent estimation of β_i can be achieved by instrumental variable techniques assuming there exist pre-determined variables specific to country i that are correlated with $\mathrm{I}(y_{it} - c_i)$ and uncorrelated with the errors u_{it}. If there are no country-specific regressors, namely if $\boldsymbol{\alpha}_1 = \boldsymbol{\alpha}_2 = \mathbf{0}$, the contagion coefficients, β_i, are not identified. In this case

$$y_{1t} = \boldsymbol{\delta}_1'\mathbf{z}_t + \beta_1 \mathrm{I}(y_{2t} - c_2) + u_{1t},$$
$$y_{2t} = \boldsymbol{\delta}_2'\mathbf{z}_t + \beta_2 \mathrm{I}(y_{1t} - c_1) + u_{2t},$$

and the observed common drivers, $\mathbf{z}_t$, cannot be used as instruments for the crisis indicators. In this case pooling of the country equations will not help either, even if the slope homogeneity assumption is imposed (namely if $\boldsymbol{\delta}_1 = \boldsymbol{\delta}_2$, and $\beta_1 = \beta_2$).

If, however, country (market) specific regressors exist, i.e. $\boldsymbol{\alpha}_i \neq 0$, $i = 1, 2$, the following instrumental variables estimator can be used. Suppose that c_1 and c_2 are known and the observations $\mathbf{y}_t$, $\mathbf{w}_t = (\mathbf{z}_t', \mathbf{x}_{1t}', \mathbf{x}_{2t}')'$, $t = 1, 2, \ldots, T$ are given and that the following conditions are met.

(i)
$$\frac{\sum_{t=1}^{T} \mathbf{w}_t \mathbf{w}_t'}{T} \xrightarrow{p} \boldsymbol{\Sigma}_{ww},$$
where $\boldsymbol{\Sigma}_{ww}$ is a (non-stochastic) positive definite matrix.

(ii) Let $\mathbf{h}_{1t} = (\mathbf{z}_t', \mathbf{x}_{1t}', \mathrm{I}(y_{2t} - c_2))'$, and $\mathbf{h}_{2t} = (\mathbf{z}_t', \mathbf{x}_{2t}', \mathrm{I}(y_{1t} - c_1))'$, and
$$\frac{\sum_{t=1}^{T} \mathbf{w}_t \mathbf{h}_{i,t}'}{T} \xrightarrow{p} \mathbf{Q}_i,$$
where $\mathbf{Q}_i$ $i = 1, 2$ are full column rank matrices and the convergence to $\mathbf{Q}_i$ is uniform.

Then the IV estimator of $\boldsymbol{\theta}_i = (\boldsymbol{\delta}_i', \boldsymbol{\alpha}_i', \beta_i)'$, defined by

$$\hat{\boldsymbol{\theta}}_i = \left(\hat{\mathbf{Q}}_i'\hat{\boldsymbol{\Sigma}}_{ww}^{-1}\hat{\mathbf{Q}}_i\right)^{-1}\hat{\mathbf{Q}}_i'\hat{\boldsymbol{\Sigma}}_{ww}^{-1}\hat{\mathbf{q}}_i$$

where

$$\hat{\mathbf{Q}}_i = \frac{\sum_{t=1}^{T} \mathbf{w}_t \mathbf{h}_{i,t}'}{T}, \quad \hat{\boldsymbol{\Sigma}}_{ww} = \frac{\sum_{t=1}^{T} \mathbf{w}_t \mathbf{w}_t'}{T}, \quad \hat{\mathbf{q}}_i = \frac{\sum_{t=1}^{T} \mathbf{w}_t y_{it}}{T},$$

is consistent for $\boldsymbol{\theta}_i$ as $T \to \infty$.[1]

The validity of these conditions needs to be checked in the case of the particular model under consideration. For example, suppose the model of interest is given by (16) and (17), and that the conditions (18) to (20) hold, and $T^{-1}\sum_{t=1}^{T} x_{2t}x_{1t}$ tends to a finite limit as $T \to \infty$. Let

$$\operatorname*{plim}_{T\to\infty} \begin{pmatrix} T^{-1}\sum_{t=1}^{T} x_{1t}^2 & T^{-1}\sum_{t=1}^{T} x_{1t}\mathrm{I}(y_{2t} - c_2) \\ T^{-1}\sum_{t=1}^{T} x_{2t}x_{1t} & T^{-1}\sum_{t=1}^{T} x_{2t}\mathrm{I}(y_{2t} - c_2) \end{pmatrix} = \mathbf{V}_1.$$

[1] The parameters of the model, including the threshold coefficients, c_1 and c_2, can also be estimated by the maximum likelihood method. This is, however, beyond the scope of the present paper.

Then α_1 and β_1 can be identified if $\mathbf{V}_1$ has a full rank. This rank condition can be investigated using the solutions (13) and (14). Although, the exact form of $\mathbf{V}_1$ depends on the way the indeterminacy of the solution is resolved in periods where $-1 < W_{it} = (\alpha_i x_{it} + u_{it} - c_i)/\beta_i \leq 0$, for $i = 1, 2$, it would nevertheless be possible to check if $\mathbf{V}_1$ is full rank without a full specification of the d_t process. For example, it suffices to postulate that d_t follows a general Bernoulli process with a probability that varies with the state variables, x_{it}, $i = 1, 2$. In the case where x_{it} and u_{it} are strictly stationary, in view of (13) and (14), it follows that y_{it}, $i = 1, 2$ are also strictly stationary, and

$$T^{-1}\sum_{t=1}^{T} x_{1t}\mathrm{I}(y_{2t} - c_2) \xrightarrow{p} \mathrm{E}\left[x_{1t}\mathrm{I}(y_{2t} - c_2)\right],$$
$$T^{-1}\sum_{t=1}^{T} x_{2t}\mathrm{I}(y_{2t} - c_2) \xrightarrow{p} \mathrm{E}\left[x_{2t}\mathrm{I}(y_{2t} - c_2)\right].$$

These results, in conjunction with the solution (13) and (14) allow us to establish the rank of $\mathbf{V}_1$ without an exact knowledge of the d_t process.

4 Contagion in a Multi-Country Setting

Consider now a sample of N countries observed over periods $t = 1, 2, \ldots, T$, some or all of which could be subject to a crisis at least for some periods over the sample period. A generalisation of (1) and (2) to the case of $N > 2$ can be written as

$$y_{it} = \boldsymbol{\delta}_i'\mathbf{z}_t + \boldsymbol{\alpha}_i'\mathbf{x}_{it} + \beta_i\sum_{j=1}^{N} w_{ij}\mathrm{I}(y_{jt} - c_j\sigma_{j,t-1}) + u_{it}, \;\; i = 1, 2, \ldots, N,$$

where the weights $w_{ij} \geq 0$ are such that $\sum_{j=1}^{N} w_{ij} = 1$, and $w_{ii} = 0$, for all i. The theoretical literature on contagion can often be cast in terms of this general formulation. For example, Allen and Gale (2000) consider a theoretical model of financial contagion where bank failures spread from one region to another under different market structures. They study $N = 4$ countries and consider three types of market structures, namely “complete”, “incomplete”, and “disconnected incomplete”. In terms of our set up these correspond to different weighting schemes as defined by the following patterns

$$\mathbf{W}_{Complete} = (w_{ij}) = \begin{pmatrix} 0 & 1/3 & 1/3 & 1/3 \\ 1/3 & 0 & 1/3 & 1/3 \\ 1/3 & 1/3 & 0 & 1/3 \\ 1/3 & 1/3 & 1/3 & 0 \end{pmatrix},$$

$$\mathbf{W}_{Incomplete} = (w_{ij}) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix},$$

and

$$\mathbf{W}_{Disconnected} = (w_{ij}) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

Notice also that the incomplete structures pre-suppose the existence of certain ordering of the regions, although no particular ordering of the regions is required under the complete market structure. Under the disconnected incomplete structure the $N = 4$ problem reduces to two separate $N = 2$ problems and their solutions do not pose any new difficulties. The incomplete market pattern can be reduced to the following generalisation of (7) and (8)

$$\begin{aligned} Y_{1t} &= W_{1t} + \mathrm{I}(Y_{2t}), \\ Y_{2t} &= W_{2t} + \mathrm{I}(Y_{3t}), \\ Y_{3t} &= W_{3t} + \mathrm{I}(Y_{4t}), \\ Y_{4t} &= W_{4t} + \mathrm{I}(Y_{1t}), \end{aligned}$$

where as before

$$Y_{it} = \frac{y_{it} - c_i\sigma_{i,t-1}}{\beta_i}, \; W_{it} = \frac{\boldsymbol{\delta}_i'\mathbf{z}_t + \boldsymbol{\alpha}_i'\mathbf{x}_{it} + u_{it} - c_i\sigma_{i,t-1}}{\beta_i}, \; i = 1, 2, 3, 4. \quad (23)$$

The solution in this case can be obtained along similar lines followed for the simple case of $N = 2$, although at the expense of much greater details. As before there will also be multiple solutions. For example, in the case where $W_{it} = 0$, two solutions are possible, namely $Y_{it}^a = 0$ and $Y_{it}^b = 1$.

Some interesting results can be obtained under the complete market structure. In this case (for a general N) we have

$$y_{it} = \boldsymbol{\alpha}'\mathbf{x}_{it} + \beta\left(\frac{\sum_{j=1, j\neq i}^{N}\mathrm{I}(y_{jt} - c_j)}{N-1}\right) + \gamma f_t + \varepsilon_{it}, \; i = 1, 2\ldots, N, \quad (24)$$

where for simplicity we have omitted the common observed effects $(\mathbf{z}_t)$, assumed all the coefficients are homogeneous and have characterised the interdependence of the errors using the single factor structure given by (25). Define the crisis indicator $\kappa_{it} = \mathrm{I}(y_{it} - c_i)$. Then,

$$\frac{\sum_{j=1, j\neq i}^{N}\mathrm{I}(y_{jt} - c_j)}{N-1} = \left(\frac{N}{N-1}\right)\bar{\kappa}_t - \frac{1}{N-1}\kappa_{it},$$

where $\bar{\kappa}_t = N^{-1} \sum_{i=1}^{N} \kappa_{it}$. Averaging (24) over $t = 1, 2, \ldots, T$, we have[2]

$$\bar{y}_t = \boldsymbol{\alpha}' \bar{\mathbf{x}}_{t-1} + \beta \bar{\kappa}_t + \gamma f_t + \bar{\varepsilon}_t.$$

Using this result in (24) to eliminate the unobserved common effect, f_t, we have

$$y_{it} = \boldsymbol{\alpha}' \mathbf{x}_{it} + \beta \left[\left(\frac{N}{N-1} \right) \bar{\kappa}_t - \frac{1}{N-1} \kappa_{it} \right] + \left(\bar{y}_t - \boldsymbol{\alpha}' \bar{\mathbf{x}}_t - \beta \bar{\kappa}_t - \bar{\varepsilon}_t \right) + \varepsilon_{it},$$
$$i = 1, 2 \ldots, N.$$

Hence

$$y_{it} - \bar{y}_t = \boldsymbol{\alpha}' \left(\mathbf{x}_{it} - \bar{\mathbf{x}}_t \right) - \beta \left(\frac{\kappa_{it} - \bar{\kappa}_t}{N-1} \right) + \left(\varepsilon_{it} - \bar{\varepsilon}_t \right).$$

In the case where N is sufficiently large, the second term converges to zero and β cannot be identified, although a consistent estimator of $\boldsymbol{\alpha}$ can be obtained from an OLS regression of $y_{it} - \bar{y}_t$ on $(\mathbf{x}_{it} - \bar{\mathbf{x}}_t)$. Allowing for parameter heterogeneity does not resolve this problem. For N fixed as $T \to \infty$, the condition for identification of β is similar to the two-country case discussed in Section 3 above.

5 A Re-examination of Existing Tests of Contagion

Using the insights gained from the canonical model we now reconsider the extant, empirical literature on contagion. We concentrate on the two most commonly used approaches: Correlation based tests of contagion and tests based on panel data analysis of currency crises.

5.1 Correlation Based Tests of Contagion

In a number of papers by Boyer, Gibson, and Loretan (1999), Loretan and English (2000), Forbes and Rigobon (2002) and Corsetti, Pericoli and Sbracia (2005) attempts have been made to identify contagion effects from pairwise correlation of stock market returns by testing whether correlation is significantly higher during crises times compared to normal periods. The main difference between these studies is in how the correlation coefficient is adjusted for the higher volatility experienced in crises periods. All these studies require *a priori* specification of the crises periods. The data employed are typically daily return observations and do not consider global or country-specific variables in their analysis.

[2]See Pesaran (2005) for a general discussion of the analysis of cross-sectional dependence in large panels.

In terms of our set up the basic model underlying this approach can be written as (following the approach of Corsetti et al.)

$$y_{1t} = \alpha_1 + \beta_1 \mathrm{I}(y_{2t} - c_{2t}) + u_{1t},$$
$$y_{2t} = \alpha_2 + \beta_2 \mathrm{I}(y_{1t} - c_{1t}) + u_{2t},$$

where guess-estimates of c_{it} are obtained from conditional sample means and standard deviations of y_{it} in an informal manner. The interdependence across the two countries is characterised using the single factor specification

$$u_{it} = \gamma_i \ f_t + \varepsilon_{it}, \tag{25}$$

where f_t is the unobserved common factor, and ε_{it}, $i = 1, 2$ are idiosyncratic shocks:

$$f_t \sim iid(0, 1),$$
$$\varepsilon_{it} \sim iid(0, \sigma_i^2).$$

f_t and ε_{it} are also assumed to be independently distributed. For the two-country set up the single factor model is algebraically equivalent to assuming u_{1t} and u_{2t} are correlated with the correlation coefficient

$$\rho = \frac{\gamma_1 \gamma_2}{\sqrt{\sigma_1^2 + \gamma_1^2}\sqrt{\sigma_2^2 + \gamma_2^2}}.$$

Under this set up there exist no valid instruments with which to identify the contagion coefficient from the interdependence coefficient ρ. The identification problem is overcome in this literature by assuming that the crisis periods are known *a priori*, and are sufficiently prolonged and continuous so that correlation of y_{1t} and y_{2t} during crisis and non-crisis periods can be consistently estimated and compared.

Therefore, this approach is problematic on three counts.

1. The endogeneity problem discussed in the previous section is circumvented by separating crises periods from non-crises periods. Since crisis periods are identified *ex post*, after passing through the observations, the endogeneity bias is re-introduced, however, in form of a sample selection bias.[3]

2. Multi-country, multi-asset (market) generalisations of the correlation/covariance approach will require existence of much longer periods of continuous crisis for the estimation and testing strategy to be meaningful. Such data sets are unlikely to exist since by their very nature crisis periods are relatively short.

[3]The problem of sample selection bias also applies to other approaches, such as that of Glick and Rose (1999) and Caramazza, Ricci and Salgado (2004), who select only crises periods to study contagion.

3. The correlation analysis cannot be used in forecasting and is of limited scope in a structural understanding of the crises and the factors behind their occurrence.

5.2 Panel Estimates of Contagion Effects

Eichengreen, Rose and Wyplosz (1996), Esquivel and Larrain (1998), Kruger, Osakwe and Page (1998), Kumar, Moorthy and Perraudin (2002) and Stone and Weeks (2001) attempt to estimate and test for contagion effects using panel data models. The econometric approach taken in these papers is based on binary choice models with linear index functions

$$y_{it} = \alpha_{0i} + \boldsymbol{\alpha}'\mathbf{x}_{it} + \varepsilon_{it}, \quad i = 1, 2, \ldots, N, \ t = 1, 2, \ldots, T, \tag{26}$$

where y_{it} is a latent variable observed qualitatively through a univariate binary response indicator, $\kappa_{it} = \mathrm{I}(y_{it})$, the currency crisis indicator, $\mathbf{x}_{it}$ is a $k \times 1$ vector of observed macroeconomic and political variables, $\boldsymbol{\alpha}$ is a $k \times 1$ vector of unknown coefficients and ε_{it} is an idiosyncratic error assumed to be serially uncorrelated for each i, and *iid* normally distributed across i with mean zero, a unit variance. Except for Esquivel and Larrain (1998), who use a random effects probit model, the literature assumes that $\alpha_{0i} = \alpha_0$.

Contagion is addressed by including a dummy variable, $\mathcal{C}_{it}$, in model (26),

$$y_{it} = \alpha_{0i} + \beta\mathcal{C}_{it} + \boldsymbol{\alpha}'\mathbf{x}_{it} + \varepsilon_{it}, \tag{27}$$

where

$$\mathcal{C}_{it} = \mathrm{I}\left(\sum_{j=1, j\neq i}^{N} \kappa_{jt}\right). \tag{28}$$

Under this formulation the crisis indicator, $\mathcal{C}_{it}$, takes the value of unity if any one of the $N-1$ remaining countries find themselves in a crisis state. This formulation is quite similar to that discussed above and is subject to similar identification and estimation problems.[4] Due to the non-linear nature of this formulation, in order to assess the impact of the endogeneity on the parameter estimates in the probit model of (26) we conduct a Monte Carlo experiment using the data of Eichengreen et al. (1996). Details of the data are given in the Appendix C.

5.2.1 Experimental Design

Simulation with artificial regressors The Monte Carlo experiments are based on the following data generating process (DGP),

$$y_{it}^{r} = \alpha_0 + \boldsymbol{\alpha}'\mathbf{x}_{it}^{r} + u_{it}^{r}, \quad i = 1, 2, \ldots, N, \ t = 1, 2, \ldots, T, \ r = 1, 2, \ldots, R,$$

[4] The problem of simultaneity also affects other approaches. Kaminsky and Reinhart (2000) add a contagion index similar to that of ERW to the macroeconomic variables on the right hand side to explain the probability of currency crises.

where r refers to the replication number in the Monte Carlo experiments, R is the total number of replications, $\mathbf{x}_{it}^r$ is a $k \times 1$ vector of simulated exogenous variables. Under this DGP, β, the contagion coefficient in (27), is set equal to zero and all other coefficients are identical across i.

The estimation of α_0 and $\boldsymbol{\alpha}$ under a probit specification only makes use of $\kappa_{it}^r = \mathrm{I}(y_{it}^r)$ and, hence, without loss of generality the variance of the error term, u_{it}^r, may be set equal to unity. To allow for correlation across the errors of different cross section units we adopt the following standardised one-factor structure

$$u_{it}^r = \frac{\gamma_i f_t^r + \varepsilon_{it}^r}{\sqrt{1+\gamma_i^2}}$$

where γ_i is a scalar, $f_t^r \sim \mathrm{iidN}(0,1)$, and $\varepsilon_{it}^r \sim \mathrm{iidN}(0,1)$. Under these assumptions we have $\mathrm{E}(u_{it}^r) = 0$ and $\mathrm{Var}(u_{it}^r) = 1$. The pairwise correlation coefficient of the errors is given by

$$\mathrm{Corr}\left(u_{it}^r, u_{jt}^r\right) = \frac{\gamma_i \gamma_j}{\sqrt{\left(1+\gamma_i^2\right)\left(1+\gamma_j^2\right)}}.$$

Treating values of $y_{it}^r > 0$ as crises, in all our experiments we fix α_0 such that the fraction of observations, ψ, with $y_{it}^r > 0$ is non-zero but relatively small, namely $\psi = 0.05$. For this purpose, assuming that the regressors are normally distributed we have $\boldsymbol{\alpha}'\mathbf{x}_{it} + u_{it} \sim \mathrm{iidN}(0, 1 + \boldsymbol{\alpha}'\Sigma_x\boldsymbol{\alpha})$ and therefore

$$\Pr\left(y_{it}^r > 0\right) = \Pr\left(\boldsymbol{\alpha}'\mathbf{x}_{it}^r + u_{it}^r > -\alpha_0\right) = 1 - \Phi\left(\frac{-\alpha_0}{\sqrt{1+\boldsymbol{\alpha}'\boldsymbol{\Sigma}_x\boldsymbol{\alpha}}}\right) = \psi.$$

Hence, we set

$$\alpha_0 = -\left(1 + \boldsymbol{\alpha}'\boldsymbol{\Sigma}_x\boldsymbol{\alpha}\right)^{1/2}\ \Phi^{-1}(1-\psi). \tag{29}$$

This is an important choice in the Monte Carlo experiment because the contagion dummy becomes a vector of ones if the proportion of crises periods is too high or a vector of zeros if the proportion of crises periods is too low. In such a case the right hand side variables are perfectly collinear as they contain an intercept and the contagion dummy.

For each replication a contagion dummy, $\mathcal{C}_{it}^r$, is constructed as

$$\mathcal{C}_{it}^r = \mathrm{I}\left(\sum_{j=1, j\neq i}^{N} \kappa_{jt}^r\right).$$

For the probit estimation only the binary indicator $\kappa_{it}^r = \mathrm{I}(y_{it}^r)$ is observed. The probability of $\kappa_{it}^r = 1$ is modelled as

$$\Pr(\kappa_{it}^r = 1) = \Phi(\alpha_0 + \beta\mathcal{C}_{it}^r + \boldsymbol{\alpha}'\mathbf{x}_{it}^r),$$

and for the linear OLS regression the assumed model is

$$y_{it}^r = \alpha_0 + \beta \mathcal{C}_{it}^r + \boldsymbol{\alpha}' \mathbf{x}_{it}^r + e_{it}^r,$$

where $e_{it}^r \sim \text{iid}(0, \sigma_e^2)$. The parameters of the probit model (in particular the contagion coefficient, β) are computed by the maximum likelihood method.

In a first set of Monte Carlo experiments, we generate $\mathbf{x}_{it}^r \sim \text{iid}(\mathbf{0}, \mathbf{\Sigma}_x)$ for $k = 2$.[5] We fix $\mathbf{\Sigma}_x$ by generating the regressors with the following common factor structure

$$x_{it}^r = \frac{1}{\sqrt{1+\phi_i}}(q_{it}^r + \phi_i h_t^r), \tag{30}$$

where $q_{it}^r \sim \text{iidN}(0,1)$, and $h_t^r \sim \text{iidN}(0,1)$. To ensure that the regressors are distributed independently of the errors, h_t^r and f_t^r are taken to be independent draws. Finally, without loss of generality we set $\boldsymbol{\alpha} = \boldsymbol{\iota}_k$, a $k \times 1$ vector of ones. Note that under $\phi_i = 0$, $\mathbf{\Sigma}_x = \mathbf{I}_k$, and using (29) we have $\alpha_0 = 1.96\,(\sqrt{1+k})$ for $\psi = 0.025$. In the case where $\phi_i > 0$, $\mathbf{\Sigma}_x$ will have typical off diagonal elements $\sigma_{ij} = \phi_i \phi_j / (\sqrt{1+\phi_i}\sqrt{1+\phi_j})$, and α_0 follows from (29).

Note that, while we appreciate that parameter heterogeneity may be important in applications to real data, we abstract from it in the Monte Carlo experiment for simplicity. Intercept heterogeneity could be introduced via a random effects probit model or a conditional logit model, see Hsiao (2003).

Simulation with ERW regressors In a second set of Monte Carlo experiments the exogenous regressors of Eichengreen et al. (1996) are used and taken as given across all the replications. Under the null of no contagion β is set equal to zero and the other parameters, $(\alpha_0, \boldsymbol{\alpha})$, are set equal to the estimates of the pooled probit model computed using the ERW data. These estimates, denoted $\hat{\alpha}_0$ and $\hat{\boldsymbol{\alpha}}$ are given in Table 2.

Hence, a vector $\mathbf{y}^r$ is generated as

$$y_{it}^r = \hat{\alpha}_0 + \hat{\boldsymbol{\alpha}}' \mathbf{x}_{it} + u_{it}^r$$

The specification of the error term and the estimation are as in the case of artificial data.

5.2.2 Results of the Monte Carlo Experiments

Results for the artificial regressors Tables 3–6 give the results for the Monte Carlo experiments with artificially generated regressors. Tables 3–4 show the results for orthogonal regressors and Tables 5–6 show the results

[5] We have also performed Monte Carlo experiments with $k = 1$, and the results for the contagion parameter are unchanged. In order to keep the presentation concise we only report the experiments with $k = 2$.

Table 2: Probit model with ERW data

variable	$(\hat{\alpha}_0, \hat{\boldsymbol{\alpha}})$	$\lvert t \rvert$-value
Intercept ($\hat{\alpha}_0$)	-1.886	10.751
Capital controls	-0.134	0.717
Government victory	-0.060	1.141
Government loss	-0.332	0.787
Credit growth	0.016	1.880
Inflation	0.065	3.584
Output growth	0.020	0.732
Employment growth	0.043	1.007
Unemployment rate	0.073	3.010
Budget position	0.042	2.042
Current account	-0.024	1.072

Total number of observations = 645

for regressors that are correlated with $\phi_i = 0.5$, $\forall i$, see (30). The first of each set of tables, Tables 3 and 5, reports the results for the discretised dependent variable, i. e. the estimates from the probit model. The second of each set of tables, Tables 4 and 6, are for the continuous dependent variable estimated via OLS.

For all experiments the bias increases with the size of the error correlation across i. For small and even medium sample sizes the estimate of β is quite imprecise in the probit model. However, the OLS estimates of the contagion effects, β, under error interdependence ($\rho = \gamma^2/(1+\gamma^2) \neq 0$) is positive in all the experiments. This confirms the upward bias derived theoretically in the context of our simple two-country canonical model.

The last panel of each table gives the rejection probability for the hypothesis of no contagion, that is the proportion of experiments where the null hypothesis $\mathrm{H}_0 : \beta = 0$ is rejected. It can be seen that the rejection probability rises as interdependence increases. With $\gamma = 1$, which is equivalent to an error correlation of 0.5, $N = T = 100$ the hypothesis of no contagion is virtually always rejected in all models. However, even mild interdependence leads to high rejection rates. In the OLS estimation with $\phi = 0$, $\gamma = 0.4$, which implies correlation of 0.14, and $N = T = 50$ the hypothesis of no contagion is rejected in 96.3% of cases.

The results show that the precision of the estimates does not improve equally when increasing N or T. In all the experiments the root mean square errors are systematically lower with T larger than N for a given number of observations NT. For example in Table 3, for $\gamma = 1$, for $T = 50$, $N = 100$, the RMSE is 1.038 and for $T = 100$, $N = 50$ it is 0.880. To understand this recall that the contagion variable is 1 for all i if there are at least two crises in the period. Hence, in such a situation the variation of the contagion index remains unchanged if other countries are added, and the effect of increasing N will be limited.

Results based on the ERW regressors Table 7 shows that both, the OLS and the probit results, produce a marked upward bias in the estimates of the contagion coefficient for non-zero values of γ and that the bias increases in γ. The bias could be substantial even for moderate degrees of cross dependence. For example, for $\gamma = 0.4$ (which corresponds to a pair-wise cross correlation coefficient of around 0.14) the pooled panel estimate of β is 0.27 as compared to its true value of zero. This result holds under both of the alternative estimation procedures.

The null hypothesis of $\beta = 0$ is also rejected well in excess of the nominal 5% level for all non-zero values of γ. The pooled probit estimates also exhibit a substantial degree of over-rejection (12.3% as compared to 5%) even under $\gamma = 0$. The degree of over-rejection of the pooled OLS estimates (7.2%) is much less pronounced, although still significantly different from 5% considering that the experiments are based on 2000 replications.

In view of these results it is reasonable to conclude that the estimate of the contagion coefficient of 0.54 that one obtains from pooled probit estimation using the ERW data could be wholly or partly due to *neglected* inter-dependencies of the equation errors across different countries.

6 Application to European Interest Rates Spreads

In this section we provide an empirical application of the model presented in this paper using data on European interest rates spreads analyzed by Favero and Giavazzi (2002).[6] The data are three month interest rates spreads for seven European countries (the Netherlands, France, Italy, Spain, Denmark, Sweden, and Belgium) with weekly observations taken on Wednesdays over the period 2 November 1988 to 9 September 1992. The canonical model presented in this paper provides a formal statistical framework for a simultaneous analysis of contagion and interdependence without an *a priori* classification of the observations into crisis and non-crisis periods.

Favero and Giavazzi (2002) consider positive as well as negative extreme movements in the spreads, and pre-identify these extreme observations based on residuals from a first stage VAR (Vector Autoregressive) analysis in the seven spreads. In our application we introduce the upside and the downside crises dummies in our canonical model and consider the equations

$$\Delta y_{it} = \alpha_{0i} + \alpha_{i1}\Delta y_{i,t-1} + \alpha_{i2}\Delta y_{i,t-2} + \beta_i^{+}\mathcal{C}_{it}^{+} + \beta_i^{-}\mathcal{C}_{it}^{-} + \varepsilon_{it}, \qquad (31)$$

where Δy_{it} is the first difference in the spreads used by Favero and Giavazzi

[6] We thank Carlo Favero for providing us with the data.

(2002). The contagion indices are defined as

$$\mathcal{C}_{it}^{+} = \mathrm{I}\left(\sum_{j=1,J\neq i}^{N} \mathrm{I}(\Delta y_{jt} - c_j)\right),$$

and

$$\mathcal{C}_{it}^{-} = \mathrm{I}\left(\sum_{j=1,J\neq i}^{N} \mathrm{I}(-\Delta y_{jt} - c_j)\right),$$

where $c_j > 0$ is set to two standard deviations of Δy_{it}, which implies that 2.9% of observations are positive crises observations and 2.1% negative crises observations. We have also tried other threshold levels and the results did not vary substantially.[7]

Equation (31) is estimated country by country using the Generalized Instrumental Variables Estimation (GIVE) procedure with the lagged dependent variables of the countries $j = 1, 2, \ldots, N$, $j \neq i$, used as instruments for $\mathcal{C}_{it}^{+}$ and $\mathcal{C}_{it}^{-}$. Given that the endogenous variables $\mathcal{C}_{it}^{+}$ and $\mathcal{C}_{it}^{-}$ are nonlinear functions of the dependent variables, the strength of the instruments can be improved by also considering power series of the instruments (Kelejian 1971, Newey 1990). We construct powers of the lagged endogenous variables

$$\mathbf{w}_{jt}(m) = [\Delta y_{j,t-1}, (\Delta y_{j,t-1})^2, \ldots, (\Delta y_{j,t-1})^m, \Delta y_{j,t-2}, (\Delta y_{j,t-2})^2, \ldots, (\Delta y_{j,t-2})^m],$$

and use

$$\mathbf{W}_{it}(m) = [\mathbf{w}_{1t}(m), \mathbf{w}_{2t}(m), \ldots, \mathbf{w}_{i-1,t}(m), \mathbf{w}_{i+1,t}(m), \ldots, \mathbf{w}_{Nt}(m)],$$

as instruments for $\mathcal{C}_{it}^{+}$ and $\mathcal{C}_{it}^{-}$. In the applications we considered powers $m = 1, 2, \ldots, 6$, which also gives an insight into the robustness of the results to the choice of m. We also investigate the weak instrument problem by reporting the Cragg-Donald statistic (Cragg and Donald 1993, Stock and Yogo 2005) for the GIVE estimates.

The results are summarized in Table 8. The top panel provides the OLS estimates (that do not take the endogeneity of the contagion indices into account). For three countries, France, Spain, and Belgium, β_i^{+} is significant at least at 5% level, and β_i^{-} is significant for all countries except Italy. The results in the subsequent panels of Table 8 provide the instrumental variable estimates using $\mathbf{W}_{it}(m)$ as the instruments. Setting $m = 1$, β_i^{+} continues to be statistically significant in the case of France, Spain, and Belgium, whereas β_i^{-} becomes statistically insignificant for all the seven spreads. Using $m = 2$ and 3 leads to the same results. When $m = 4$ the contagion coefficient, β_i^{+}

[7]Another possible option would have been to include the Δy_{jt}, $j = 1, 2, \ldots N$, $j \neq i$ in the right hand side of the regression. However, this model lead to a loss in power in the estimation and all coefficients were inconsistent.

in the equation for Italy becomes also significant, and when $m = 5$, β_i^+ in the equation for Spain becomes insignificant, and the same results applies to $m = 6$.

Overall, the test results provide some evidence of contagion. But the effects are asymmetric, with no significant effects from sharp declines in the spreads, contrary to the OLS estimates. The statistical significance of the results should also be viewed with some caution, since the Cragg-Donald statistics reported in Table 8 show that the null of weak instruments cannot be rejected (Stock and Yogo 2005). Nevertheless, the statistical insignificance of β_i^- irrespective of the order of the power augmentation of the instruments (m), suggests that the significance of the OLS estimates of β_i^-, is most likely due to interdependence rather than contagion.

7 Conclusions

In this paper we have developed a canonical model of contagion. Using this model, we have considered the issue of identification and consistent estimation of contagion coefficients. We show that in the presence of error inter-dependencies contagion effects cannot be consistently estimated without country-specific regressors. This clearly highlights some of the pitfalls that surround the empirical studies of currency crises and financial contagion that are extant in the literature. Correlation analyses look for significant shifts in correlation coefficients across crises and tranquil periods without the use of country specific variables. In the case of such data sets identification of contagion is achieved by making strong *a priori* assumptions concerning sample splits into "crisis" and "no-crisis" periods. Invariably, this also involves the identification of the source country in which the crisis is purported to have begun.

Multi-country panel analyses of the type carried out by ERW do contain country specific fundamentals and could in principle be used to shed light on the issue of contagion versus interdependence. However, panel data studies are typically carried out assuming that contagion indices are predetermined and that equation errors across countries/markets are independently distributed, and as we have shown this could introduce a substantial upward bias in the estimates of the contagion coefficients.

The canonical model presented in this paper provides a formal statistical framework for a simultaneous analysis of contagion and interdependence without an a priori classification of the observations into crisis and non-crisis periods. This is illustrated using the data on European interest rates spreads analyzed by Favero and Giavazzi (2002). We find that contagion indices corresponding to sharp falls in the spreads (measured relative to the German interest rate) that are significant when using OLS become insignificant when accounting for their endogeneity using instrumental variables.

However, the statistical significance of sharp rises in the spreads for some of the European countries (France, Spain and Belgium) continue to remain statistically significant even after instrumentation. Not withstanding the possible weak instrument problem, these results provide some evidence of contagion in the transmission of interest rate shocks across the European bond markets during 1988-1992 (ERM period), but only when the interest rates rise relative to the German interest rate and not the reverse.

Appendix A: Further Mathematical Results

Lemma 2 *Suppose* $\mathbf{x}_{it}$ *and* u_{it}*, for* $i = 1, 2$*, are serially uncorrelated random variables and the joint probability density of* (u_{1t}, u_{2t}) *has positive support over* $\mathbb{R}^2$*, then*

$$\frac{1}{T}\sum_{t=1}^{T} \mathrm{I}(y_{it} - c_i) \xrightarrow{p} \pi_i, \text{ as } T \to \infty,$$

and $0 < \pi_i < 1$.

Proof. The $\mathrm{I}(y_{it} - c_i)$ are binary, *iid* random variables with $\Pr(\mathrm{I}(y_{it} - c_i)) = 1$ resulting from (13), and the sample mean will converge to the expectation, which we now show to lie between 0 and 1. We have that

$$\begin{aligned}
\mathrm{E}(\mathrm{I}(y_{it} - c_i)) &= \Pr(Y_{it} > 0) \\
&= \Pr(W_{it} + 1 > 0 | W_{jt} > 0) \Pr(W_{jt} > 0) \\
&\quad + \Pr(W_{it} + 1 > 0 | W_{it} > 0, -1 < W_{jt} \leq 0) \\
&\quad \times \Pr(W_{it} > 0, -1 < W_{jt} \leq 0) \\
&\quad + \Pr(W_{it} > 0 | W_{jt} \leq -1) \Pr(W_{jt} \leq -1) \\
&\quad + \Pr(W_{it} > 0 | W_{it} \leq -1, -1 < W_{jt} \leq 0) \\
&\quad \times \Pr(W_{it} \leq -1, -1 < W_{jt} \leq 0) \\
&\quad + \Pr(Y^*(d) > 0 | -1 < W_{it} \leq 0, -1 < W_{jt} \leq 0) \\
&\quad \times \Pr(-1 < W_{it} \leq 0, -1 < W_{jt} \leq 0) \\
&= \Pr(W_{it} + 1 > 0 | W_{jt} > 0) \Pr(W_{jt} > 0) \\
&\quad + \Pr(W_{it} > 0) \Pr(W_{jt} \leq 0 | W_{it} > 0) \\
&\quad + (1 - \pi_d) \Pr(-1 < W_{it} \leq 0, -1 < W_{jt} \leq 0) \\
&\equiv \pi_i.
\end{aligned}$$

If the joint distribution of u_{1t} and u_{2t} and, therefore, also that of W_{1t} and W_{2t} has positive support over $\mathbb{R}^2$, then at least $\Pr(W_{it} + 1 > 0 | W_{jt} > 0) \neq 0$ and $\Pr(W_{jt} > 0) \neq 0$. Hence, $\pi_i > 0$.

In order to see that $\pi_i < 1$ consider

$$\begin{aligned}
\Pr(Y_{it} > 0) &= 1 - \Pr(Y_{it} \leq 0) \\
&= 1 - \Pr(W_{it} + 1 \leq 0 | W_{jt} > 0) \Pr(W_{jt} > 0) \\
&\quad + \Pr(W_{it} \leq 0 | W_{jt} \leq -1) \Pr(W_{jt} \leq -1) \\
&\quad + \Pr(-1 < W_{jt} \leq 0) \Pr(W_{it} \leq -1) \\
&\quad + \pi_d \Pr(-1 < W_{it} \leq 0, -1 < W_{jt} \leq 0).
\end{aligned}$$

Again, if the joint distribution of u_{1t} and u_{2t} and, therefore, also that of W_{1t} and W_{2t} has positive support over $\mathbb{R}^2$, then at least $\Pr(W_{it} + 1 \leq 0 | W_{jt} > 0) \neq 0$ and $\Pr(W_{jt} > 0) \neq 0$, which is sufficient to ensure that $\pi_i < 1$ ■

Note that the assumption that $\mathbf{x}_{it}$ and u_{it} are serially uncorrelated is made for expositional convenience and strictly speaking not necessary.

Appendix B: Simulation of $\mathrm{E}(u_{2t}\mathrm{I}(y_{1t} - c_1))$

Table A reports the simulated values of $\mathrm{E}\left[u_{2t}\mathrm{I}(y_{1t} - c_1)\right]$ using the sample equivalent $\sum_{t=1}^{T}\left[u_{2t}\mathrm{I}(y_{1t} - c_1)\right]/T$ with $T = 2,000,000$. The data are generated from the reduced form of the model given by Equations (13) and (14) with $k = 1$, $x_{it}, u_{it} \sim \mathrm{iid}\,\mathrm{N}(0, 1)$, $\Pr(d_t = 1) = 0.50$, and $c_i = 1.64$. It can be seen that only for $\rho = \beta = 0$ the simulated value is zero. Similar results are also obtained for other choices of the solution indicator, d_t, namely $d_t = 0$, or $d_t = 1$.

Appendix C: Details of the ERW Data Set

The data set used by Eichengreen et al. (1996) is available on the internet at `http://haas.berkeley.edu/~arose/RecRes.htm` along with a *Stata* log file.

"The data set is quarterly, spanning 1959 through 1993 for twenty industrial countries." (Eichengreen et al. 1996, p. 477) The countries are the USA, UK, Austria, Belgium, Denmark, France, Italy, Netherlands, Norway, Sweden, Switzerland, Canada, Japan, Finland, Greece, Ireland, Portugal, Spain, Australia and Germany as the centre country. "Most of the variables are transformed into differential percentage changes by taking differences between domestic and German annualised fourth-differences of natural logarithms and multiplying by a hundred." (Eichengreen et al. 1996, p. 477).

The variables are: Total non-gold international reserves (IMF IFS line 1ld), exchange rate with US dollar (rf), money market rates (60b) or where unavailable discount rates (60), exports and imports (70 and 71), the current account (80) and the central governments budget position (80) both as percentages of nominal GDP (99a), long term bond yields (61), nominal stock market index (62), domestic credit (32), M1 (34), M2 (35 + M1), CPI (64), real GDP (99a.r), and relative unit labour cost (reu). Further from the OECD's Main Economic Indicators employment and unemployment, and Eichengreen et al. construct "indicators of government electoral victories and defeats, using Keesing's *Record of World Events* and Banks' *Political Handbook of the World*." (Eichengreen et al. 1996, p. 477)

Eichengreen et al. use the following definition of the exchange-rate market pressure index

$$EMP_{it} = \lambda_1 \%\Delta e_{it} + \lambda_2 \%\Delta(r_{it} - r_{Gt}) - \lambda_3(\%\Delta f_{it} - \%\Delta f_{Gt}), \qquad (32)$$

where e_{it} is the exchange rate to the US Dollar, r_{it} the interest rate, and f_{it} the international reserves of country i. Subscript G indicates variables for

Table A: Simulated Values of $\sum_{t=1}^{T}[u_{2t}\mathrm{I}(y_{1t}-c_1)]/T$

ρ	β	α				
		-4	-1	0	1	4
-0.99	-4	-0.095	-0.143	-0.103	-0.142	-0.096
	-1	-0.092	-0.143	-0.103	-0.142	-0.092
	0	-0.089	-0.142	-0.103	-0.143	-0.088
	1	-0.082	-0.140	-0.103	-0.140	-0.083
	4	-0.054	-0.037	-0.024	-0.039	-0.053
-0.50	-4	-0.056	-0.077	-0.052	-0.077	-0.056
	-1	-0.050	-0.075	-0.052	-0.075	-0.050
	0	-0.045	-0.072	-0.052	-0.072	-0.044
	1	-0.036	-0.051	-0.040	-0.051	-0.037
	4	-0.011	0.030	0.023	0.031	-0.011
0.00	-4	-0.018	-0.017	-0.005	-0.017	-0.017
	-1	-0.007	-0.010	-0.004	-0.010	-0.008
	0	0.000	0.000	-0.000	0.000	-0.000
	1	0.008	0.040	0.045	0.041	0.008
	4	0.032	0.089	0.060	0.090	0.032
0.50	-4	0.022	0.035	0.026	0.035	0.021
	-1	0.036	0.053	0.036	0.052	0.036
	0	0.045	0.072	0.052	0.072	0.045
	1	0.055	0.128	0.135	0.128	0.055
	4	0.075	0.134	0.082	0.134	0.074
0.99	-4	0.060	0.078	0.010	0.078	0.060
	-1	0.078	0.112	0.047	0.112	0.079
	0	0.089	0.142	0.103	0.142	0.089
	1	0.099	0.208	0.213	0.207	0.099
	4	0.114	0.165	0.070	0.166	0.115

The results are from data generated according to equations (13) and (14), with $k = 1$, $x_{it}, u_{it} \sim \text{iid}\,\mathrm{N}(0, 1)$, $\Pr(d_t = 1) = 0.5$, $c_i = 1.64$, and $T = 2,000,000$.

Germany, which is taken as the center country. Eichengreen et al. (1996, pp.476) say that they "weight the components so as to equalize the volatility of the three components". This is accomplished by setting $\lambda_i = 1/\sigma_i$, where σ_i is the standard deviation of component i. For this data set $\sigma_1 = 0.243$, $\sigma_2 = 0.037$, and $\sigma_3 = 0.0047$.

The crisis index is the calculated as

$$y_{it} = \begin{cases} 1 & EMP_{it} > \mu_{EMP} + 1.5\sigma_{EMP} \\ 0 & \text{otherwise} \end{cases}$$

where μ_{EMP} is the mean and σ_{EMP} is the standard deviation of the exchange rate market pressure index.

The credit growth, the inflation rate, the output growth and the current account are calculated as

$$dx_{it} = 100 * \ln(x_{it}/x_{it-4}) - \ln(x_{Gt}/x_{Gt-4}), \tag{33}$$

where x_{it} is the variable for country i and Germany, G. The relative unemployment rate is $dx_{it} = x_{it} - x_{Gt}$. The relative budget position is defined as $db_{it} = b_{it}/y_{it} - b_{Gt}/y_{Gt}$, where b_{it} is the nominal government budget of country i, y_{it} is the GDP of country i and Germany, G. The dummies for capital controls, government electoral victory and government electoral loss are not transformed. The other variables mentioned above are not used.

"To avoid counting the same crisis more than once, we exclude the later observation(s) when two (or more) crises occur in successive quarters." (Eichengreen et al. 1996, p.476) Country by country excluding time periods with missing data results in 645 observations for 17 countries with 56 crises observations. The countries are the USA, the UK, Austria, Belgium, Denmark, France, Germany, Italy, the Netherlands, Norway, Canada, Japan, Finland, Greece, Ireland, Portugal, Spain, and Australia.

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Table 3: Bias, RMSE, and Power of the Contagion Coefficient in a Probit Model ($\phi_i = 0$)

γ	$T =$	20				50				100		
	$N = 10$	20	50	100	10	20	50	100	10	20	50	100
						Bias						
0	−1.996	−0.608	−0.029	0.136	−1.108	−0.071	−0.011	0.076	−0.316	−0.015	−0.011	0.014
0.2	−1.914	−0.471	0.051	0.224	−0.951	−0.004	0.075	0.170	−0.217	0.042	0.081	0.132
0.4	−1.564	−0.279	0.256	0.399	−0.659	0.166	0.273	0.370	−0.038	0.194	0.272	0.368
0.6	−1.094	0.049	0.495	0.624	−0.334	0.367	0.502	0.598	0.212	0.402	0.496	0.607
0.8	−0.733	0.248	0.691	0.835	−0.097	0.573	0.688	0.812	0.413	0.581	0.683	0.804
1	−0.563	0.438	0.899	1.003	0.155	0.730	0.850	0.993	0.585	0.743	0.852	0.981
						RMSE						
0	3.165	1.822	0.466	0.684	2.297	0.563	0.261	0.291	1.161	0.262	0.183	0.218
0.2	3.159	1.683	0.468	0.647	2.164	0.590	0.275	0.340	1.130	0.263	0.201	0.260
0.4	2.960	1.635	0.537	0.710	1.974	0.517	0.384	0.476	1.015	0.345	0.332	0.424
0.6	2.835	1.536	0.711	0.868	1.818	0.581	0.580	0.666	0.888	0.486	0.534	0.641
0.8	2.697	1.614	0.873	1.040	1.765	0.761	0.752	0.865	0.930	0.644	0.713	0.830
1	2.651	1.730	1.093	1.146	1.688	0.875	0.906	1.038	0.956	0.798	0.880	1.003
						Rejection probability						
0	0.078	0.102	0.110	0.136	0.087	0.116	0.123	0.173	0.097	0.112	0.114	0.151
0.2	0.086	0.123	0.156	0.205	0.117	0.142	0.197	0.294	0.121	0.163	0.251	0.336
0.4	0.121	0.182	0.286	0.385	0.177	0.254	0.455	0.601	0.228	0.351	0.641	0.764
0.6	0.183	0.277	0.487	0.599	0.269	0.450	0.767	0.865	0.374	0.663	0.932	0.973
0.8	0.245	0.395	0.647	0.760	0.363	0.659	0.910	0.966	0.562	0.851	0.986	0.998
1	0.295	0.458	0.754	0.872	0.458	0.756	0.967	0.993	0.655	0.940	0.998	0.999

Data are generated from $y_{it}^r = \alpha_0 + \boldsymbol{\alpha}'\mathbf{x}_{it}^r + u_{it}^r$, where $\mathbf{x}_{it}^r = \frac{1}{\sqrt{1+\phi_i}}(q_{it}^r + \phi_i \mathbf{h}_t^r)$, $\mathbf{h}_t^r, q_{it}^r \sim iid\,\mathrm{N}(0,1)$, $\boldsymbol{\alpha}$ is a vector of ones, and $\alpha_0 = -1.96\sqrt{1+\boldsymbol{\alpha}'\Sigma\boldsymbol{\alpha}}$. $u_{it} = \frac{1}{\sqrt{1+\gamma_i^2}}(\gamma_i f_t^r + \varepsilon_{it}^r)$, where $\gamma_i \sim \mathrm{U}(\frac{1}{2}\gamma, \frac{3}{2}\gamma)$, $f_t^r, \varepsilon_{it}^r \sim iid\,\mathrm{N}(0,1)$, where $\mathrm{U}(a,b)$ denotes the Uniform distribution with lower limit a and upper limit b. The probit estimations use a discretised dependent variable, $\kappa_{it}^r = \mathrm{I}(y_{it}^r)$. For the estimations, a spurious contagion dummy was added and the common factor was ignored. The results in the table are for the contagion coefficient, $\hat{\beta}$. The table reports the bias, which is given as $\sum_{r=1}^{R}(\hat{\beta}^{(r)} - \beta^0)/R$, the root mean square error, which is defined as $(\sum_{r=1}^{R}(\hat{\beta}^{(r)} - \beta^0)^2/R)^{1/2}$, where the true value $\beta^0 = 0$ in the DGP and $r = 1, 2, \ldots, R$ with $R = 2000$ is the number of replications, and finally, the the one-sided rejection probability, which is defined as the probability that the t-value is larger than the 95% critical value (1.645).

Table 4: Bias, RMSE, and Power of the Contagion Coefficient in an OLS Model ($\phi_i = 0$)

	$T =$	20			50				100			
γ	$N = 10$	20	50	100	10	20	50	100	10	20	50	100
						Bias						
0	−0.006	−0.002	−0.010	−0.000	−0.002	0.002	−0.004	0.002	0.001	−0.002	−0.001	−0.001
0.2	0.055	0.065	0.081	0.124	0.061	0.066	0.083	0.122	0.059	0.064	0.086	0.124
0.4	0.207	0.212	0.264	0.348	0.198	0.212	0.267	0.347	0.200	0.216	0.268	0.354
0.6	0.370	0.394	0.448	0.558	0.359	0.390	0.469	0.554	0.364	0.396	0.461	0.556
0.8	0.540	0.550	0.617	0.712	0.521	0.563	0.616	0.712	0.530	0.552	0.622	0.715
1	0.675	0.675	0.754	0.841	0.657	0.688	0.754	0.842	0.671	0.690	0.750	0.842
						RMSE						
0	0.182	0.110	0.081	0.086	0.118	0.068	0.048	0.052	0.081	0.048	0.032	0.040
0.2	0.215	0.158	0.147	0.206	0.148	0.110	0.110	0.156	0.108	0.086	0.100	0.143
0.4	0.332	0.288	0.318	0.414	0.262	0.245	0.289	0.374	0.235	0.233	0.280	0.367
0.6	0.484	0.462	0.498	0.616	0.412	0.418	0.487	0.575	0.392	0.412	0.471	0.566
0.8	0.645	0.615	0.663	0.763	0.571	0.592	0.636	0.732	0.554	0.566	0.631	0.725
1	0.775	0.741	0.801	0.888	0.705	0.714	0.773	0.860	0.694	0.703	0.759	0.851
						Rejection probability						
0	0.053	0.049	0.046	0.070	0.052	0.060	0.056	0.066	0.056	0.057	0.053	0.071
0.2	0.127	0.219	0.389	0.507	0.160	0.305	0.561	0.683	0.204	0.424	0.746	0.840
0.4	0.354	0.574	0.808	0.861	0.521	0.798	0.963	0.971	0.714	0.954	0.997	0.998
0.6	0.604	0.828	0.938	0.960	0.807	0.967	0.998	0.999	0.949	0.998	1.000	1.000
0.8	0.770	0.913	0.987	0.983	0.918	0.996	1.000	1.000	0.992	1.000	1.000	1.000
1	0.857	0.952	0.994	0.997	0.965	0.999	1.000	1.000	0.998	1.000	1.000	1.000

See the footnote of Table 3 for details. The only difference here is that the OLS estimations uses the continuous dependent variable, y^r_{it}.

Table 5: Bias, RMSE, and Power of the Contagion Coeffient in a Probit Model ($\phi_i = 0.5$)

	$T =$	20				50				100		
γ	$N = 10$	20	50	100	10	20	50	100	10	20	50	100
						Bias						
0	−1.231	−0.234	−0.023	−0.007	−0.324	−0.027	−0.010	−0.006	−0.072	−0.015	−0.001	−0.014
0.2	−1.110	−0.141	0.060	0.123	−0.269	0.028	0.068	0.067	0.009	0.052	0.078	0.091
0.4	−0.782	−0.014	0.268	0.304	−0.030	0.184	0.251	0.293	0.144	0.213	0.254	0.306
0.6	−0.495	0.228	0.470	0.529	0.169	0.378	0.475	0.532	0.337	0.400	0.457	0.517
0.8	−0.205	0.429	0.635	0.771	0.382	0.549	0.640	0.726	0.511	0.564	0.644	0.729
1	0.135	0.616	0.850	0.966	0.494	0.724	0.800	0.893	0.663	0.704	0.802	0.898
						RMSE						
0	2.613	1.137	0.487	0.648	1.261	0.327	0.253	0.291	0.548	0.219	0.175	0.200
0.2	2.605	1.084	0.480	0.715	1.198	0.331	0.263	0.292	0.435	0.225	0.193	0.213
0.4	2.507	1.220	0.610	0.721	1.089	0.392	0.365	0.402	0.430	0.315	0.313	0.360
0.6	2.327	1.125	0.764	0.816	1.037	0.544	0.555	0.604	0.543	0.468	0.496	0.557
0.8	2.321	1.318	0.912	1.049	1.024	0.664	0.710	0.782	0.667	0.619	0.677	0.756
1	2.372	1.505	1.096	1.246	1.165	0.843	0.865	0.943	0.781	0.756	0.833	0.923
						Rejection probability						
0	0.071	0.082	0.094	0.082	0.103	0.099	0.103	0.128	0.105	0.096	0.118	0.113
0.2	0.089	0.125	0.148	0.145	0.118	0.136	0.169	0.192	0.151	0.162	0.235	0.271
0.4	0.139	0.198	0.282	0.298	0.216	0.291	0.441	0.494	0.254	0.431	0.615	0.716
0.6	0.199	0.292	0.465	0.523	0.329	0.504	0.748	0.820	0.488	0.720	0.906	0.948
0.8	0.271	0.397	0.584	0.687	0.465	0.678	0.872	0.950	0.670	0.883	0.989	0.995
1	0.330	0.496	0.710	0.799	0.549	0.803	0.944	0.988	0.773	0.951	0.998	1.000

See footnote of Table 3.

Table 6: Bias, RMSE, and Power of the Contagion Coefficient in an OLS Model ($\phi_i = 0.5$)

	$T =$	20			50				100			
γ	$N = 10$	20	50	100	10	20	50	100	10	20	50	100
						Bias						
0	−0.006	−0.005	−0.003	−0.005	0.001	0.002	−0.001	−0.002	0.001	−0.003	−0.001	−0.002
0.2	0.052	0.059	0.069	0.081	0.054	0.061	0.069	0.080	0.058	0.061	0.072	0.082
0.4	0.198	0.212	0.237	0.256	0.206	0.208	0.230	0.263	0.202	0.210	0.235	0.262
0.6	0.378	0.374	0.411	0.437	0.364	0.383	0.416	0.446	0.368	0.387	0.412	0.446
0.8	0.542	0.521	0.558	0.604	0.536	0.540	0.575	0.599	0.524	0.541	0.568	0.605
1	0.667	0.667	0.684	0.723	0.653	0.670	0.694	0.728	0.667	0.670	0.699	0.732
						RMSE						
0	0.183	0.114	0.076	0.069	0.114	0.068	0.044	0.040	0.081	0.050	0.032	0.028
0.2	0.218	0.158	0.136	0.156	0.140	0.108	0.098	0.112	0.110	0.087	0.088	0.098
0.4	0.320	0.291	0.297	0.327	0.265	0.242	0.256	0.288	0.234	0.227	0.247	0.275
0.6	0.483	0.448	0.462	0.497	0.413	0.415	0.436	0.469	0.394	0.403	0.423	0.458
0.8	0.648	0.594	0.612	0.660	0.580	0.567	0.596	0.620	0.548	0.555	0.579	0.615
1	0.767	0.739	0.739	0.776	0.699	0.700	0.716	0.748	0.690	0.686	0.709	0.743
						Rejection probability						
0	0.047	0.050	0.054	0.060	0.061	0.056	0.047	0.041	0.061	0.051	0.060	0.049
0.2	0.120	0.208	0.345	0.459	0.153	0.294	0.486	0.604	0.223	0.399	0.675	0.761
0.4	0.336	0.564	0.754	0.788	0.551	0.782	0.921	0.954	0.738	0.936	0.993	0.997
0.6	0.615	0.792	0.920	0.935	0.823	0.961	0.997	0.996	0.959	0.998	1.000	1.000
0.8	0.765	0.895	0.970	0.972	0.947	0.994	1.000	1.000	0.994	1.000	1.000	1.000
1	0.853	0.941	0.984	0.991	0.972	0.997	0.999	1.000	0.999	1.000	1.000	1.000

See footnote of Tables 3 and 4.

Table 7: Bias, RMSE, Power of the Contagion Coefficient (ERW Data)

	Probit			OLS		
γ	Bias	RMSE	$[t > c]$	Bias	RMSE	$[t > c]$
0	−0.012	0.245	0.123	−0.005	0.095	0.072
0.2	0.069	0.247	0.212	0.079	0.127	0.289
0.4	0.282	0.375	0.535	0.276	0.300	0.887
0.6	0.528	0.588	0.858	0.492	0.510	0.996
0.8	0.774	0.822	0.977	0.696	0.711	1.000
1	0.998	1.042	0.996	0.863	0.875	1.000

Data are generated from $y^r_{it} = \hat{\alpha}_0 + \hat{\boldsymbol{\alpha}}'\mathbf{x}_{it} + \varepsilon^r_{it}$, where $\mathbf{x}_{it}$ are the data of ERW, and $\hat{\alpha}_0$ and $\hat{\boldsymbol{\alpha}}$ the respective probit estimates of the parameters. $\varepsilon^r_{it} = \gamma^r_i f^r_t + u^r_{it}$, where $\gamma^r_i \sim \mathrm{U}\left(\frac{1}{2}\gamma, \frac{3}{2}\gamma\right)$, $f^r_t, u^r_{it} \sim iid\,\mathrm{N}(0,1)$. The probit estimations use a discretised dependent variable, $\kappa^r_{it} = \mathrm{I}(y^r_{it})$, and the OLS estimations the continuous dependent variable, y^r_{it}. For the estimations, a spurious contagion dummy was added and the common factor was ignored. The results in the table are for the contagion coefficient, $\hat{\beta}$. Reported are the bias, the root mean square error, and the one-sided rejection probability denoted $[t > c]$, which are defined in the footnote of Table 3.

Table 8: OLS and GIVE Estimates of the Contagion Coefficients in the Interest Rates Spreads Equations

	NL	FR	IT	ES	DK	SW	BG
				OLS			
β^+	0.046	0.098	0.128	0.165	0.025	0.056	0.104
t	1.862	3.242	1.604	4.090	0.437	0.705	3.729
β^-	−0.064	−0.090	−0.097	−0.088	−0.178	−0.185	−0.080
t	2.541	2.792	1.126	1.934	2.908	2.229	2.683
				GIVE, $m = 1$			
β^+	−0.160	0.230	0.109	0.310	−0.471	−0.036	0.291
t	1.274	1.678	0.277	1.501	1.735	0.107	1.923
β^-	−0.142	−0.017	0.294	−0.114	0.178	0.409	0.038
t	0.922	0.130	0.973	0.798	0.594	1.189	0.325
g	0.463	0.798	0.639	0.547	0.987	1.150	0.658
				GIVE, $m = 2$			
β^+	−0.059	0.171	0.032	0.051	−0.329	−0.335	0.191
t	0.851	2.174	0.142	0.475	1.640	1.533	2.134
β^-	0.022	−0.070	0.008	0.016	0.146	0.370	0.016
t	0.366	0.878	0.038	0.175	0.851	1.532	0.256
g	1.136	1.044	1.016	1.081	0.862	1.117	0.828
				GIVE, $m = 3$			
β^+	−0.024	0.126	0.052	0.123	−0.332	−0.079	0.180
t	0.443	1.997	0.303	1.438	2.398	0.471	2.903
β^-	0.002	−0.092	−0.005	−0.018	0.041	0.259	−0.024
t	0.030	1.394	0.025	0.222	0.294	1.276	0.457
g	1.166	0.987	1.005	1.117	1.134	0.901	1.130
				GIVE, $m = 4$			
β^+	−0.010	0.122	0.083	0.166	−0.240	−0.007	0.163
t	0.224	2.356	0.542	2.224	2.079	0.054	3.118
β^-	−0.007	−0.062	−0.043	−0.057	−0.113	−0.024	−0.053
t	0.158	1.126	0.272	0.838	0.992	0.143	1.081
g	1.278	1.251	1.086	1.153	1.098	1.007	1.189
				GIVE, $m = 5$			
β^+	0.009	0.099	0.183	0.145	−0.148	0.118	0.171
t	0.225	2.117	1.361	2.113	1.474	0.908	3.701
β^-	−0.022	−0.054	−0.066	−0.061	−0.115	−0.080	−0.046
t	0.506	1.022	0.445	0.915	1.088	0.518	0.988
g	1.213	1.258	1.116	1.114	1.098	0.869	1.224
				GIVE, $m = 6$			
β^+	0.017	0.079	0.161	0.148	−0.129	0.155	0.170
t	0.472	1.805	1.275	2.330	1.386	1.268	4.045
β^-	−0.039	−0.028	−0.033	−0.014	−0.136	−0.145	−0.037
t	0.966	0.568	0.241	0.232	1.391	1.018	0.834
g	1.094	1.196	1.087	1.115	1.027	0.853	1.292

The source of the intererst rates spreads is Favero and Giavazzi (2002). The countries are The Netherlands (NL), France (FR), Italy (IT), Spain (SP), Denmark (DK), Sweden (SW), and Belgium (BG). t denotes the absolute t-value, g the Cragg-Donald statistic, and m the maximum power for the polynomial approximation of the instruments.